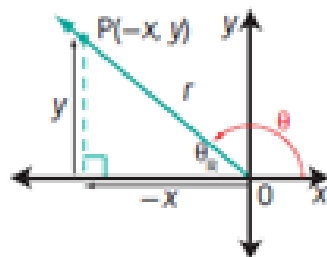


2.2 Trigonometric Ratios of Any Angle

Finding the Trigonometric Ratios of Any Angle θ , where $0^\circ \leq \theta < 360^\circ$

Quadrant II
 $90^\circ < \theta < 180^\circ$

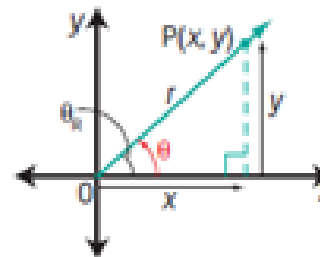
$$\begin{aligned} \sin \theta &= \frac{y}{r} & \cos \theta &= \frac{-x}{r} & \tan \theta &= \frac{y}{-x} \\ \sin \theta &> 0 & \cos \theta &< 0 & \tan \theta &< 0 \end{aligned}$$



$$\theta = 180^\circ - \theta_r$$

Quadrant I
 $0^\circ < \theta < 90^\circ$

$$\begin{aligned} \sin \theta &= \frac{y}{r} & \cos \theta &= \frac{x}{r} & \tan \theta &= \frac{y}{x} \\ \sin \theta &> 0 & \cos \theta &> 0 & \tan \theta &> 0 \end{aligned}$$

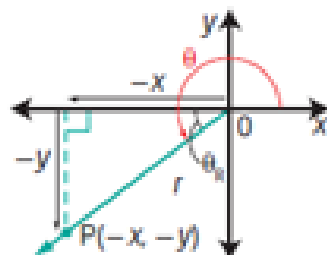


$$\theta = \theta_r$$

Why is r always positive?

Quadrant III
 $180^\circ < \theta < 270^\circ$

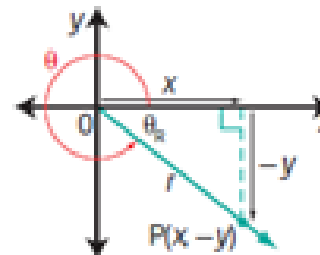
$$\begin{aligned} \sin \theta &= \frac{-y}{r} & \cos \theta &= \frac{-x}{r} & \tan \theta &= \frac{-y}{-x} \\ \sin \theta &< 0 & \cos \theta &< 0 & \tan \theta &> 0 \end{aligned}$$



$$\theta = 180^\circ + \theta_r$$

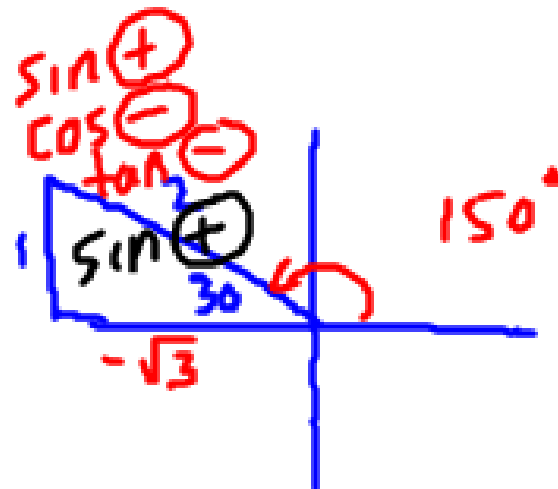
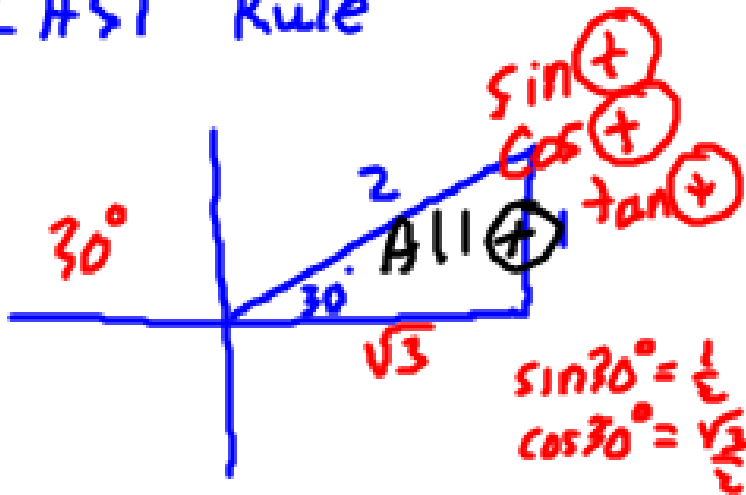
Quadrant IV
 $270^\circ < \theta < 360^\circ$

$$\begin{aligned} \sin \theta &= \frac{-y}{r} & \cos \theta &= \frac{x}{r} & \tan \theta &= \frac{-y}{x} \\ \sin \theta &< 0 & \cos \theta &> 0 & \tan \theta &< 0 \end{aligned}$$

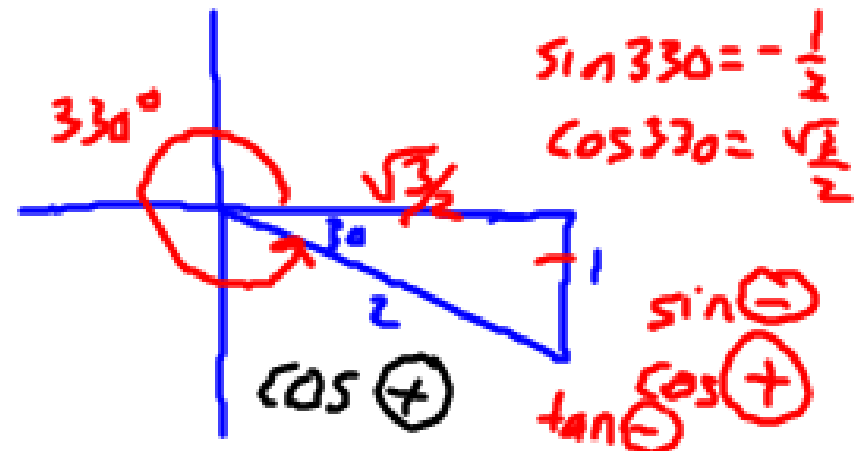


$$\theta = 360^\circ - \theta_r$$

CAST Rule



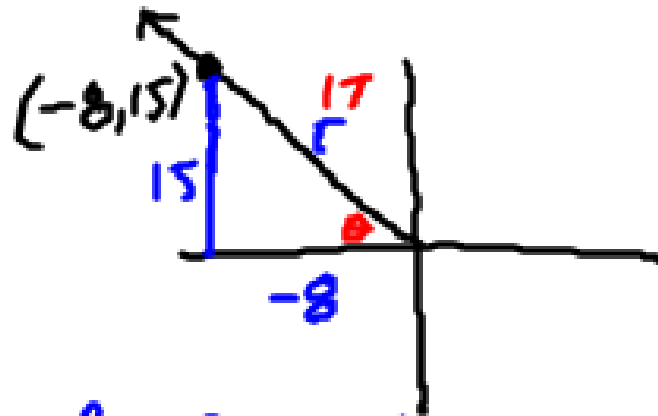
S	A
T	C



Example 1

Write Trigonometric Ratios for Angles in Any Quadrant

The point $P(-8, 15)$ lies on the terminal arm of an angle, θ , in standard position. Determine the exact trigonometric ratios for $\sin \theta$, $\cos \theta$, and $\tan \theta$.



$$r^2 = 15^2 + (-8)^2$$

$$r^2 = 225 + 64$$

$$r^2 = 289$$

$$r = 17$$

$$\sin \theta = \frac{15}{17}$$

$$\cos \theta = -\frac{8}{17}$$

$$\tan \theta = -\frac{15}{8}$$

$$\sin \theta = \frac{y}{r}$$

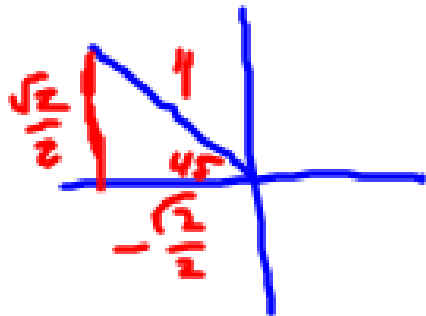
$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$

Example 2

Determine the Exact Value of a Trigonometric Ratio

Determine the exact value of $\cos 135^\circ$.

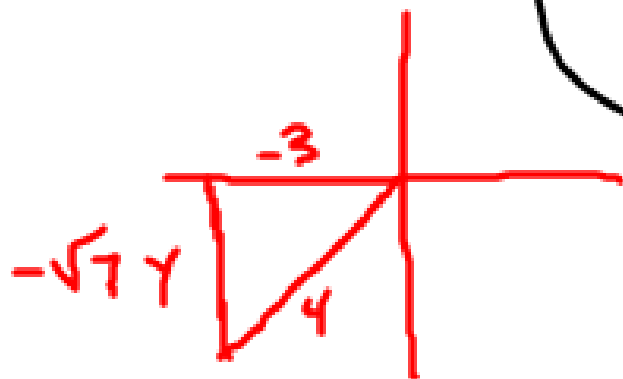


$$\cos 135^\circ = -\frac{\sqrt{2}}{2}$$

Example 3

Determine Trigonometric Ratios

Suppose θ is an angle in standard position with terminal arm in quadrant III, and $\cos \theta = -\frac{3}{4}$. What are the exact values of $\sin \theta$ and $\tan \theta$?



$$(-3)^2 + y^2 = 4^2$$

$$9 + y^2 = 16$$

$$y^2 = 7$$

$$y = \sqrt{7}$$

adj
hyp

x
↑

$$\sin \theta = -\frac{\sqrt{7}}{4}$$

$$\tan \theta = \frac{\sqrt{7}}{3}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin^2 \theta + \left(-\frac{3}{4}\right)^2 = 1$$

$$\sin^2 \theta + \frac{9}{16} = \frac{16}{16}$$

$$\sin^2 \theta = \frac{7}{16}$$

$$\sin \theta = \pm \frac{\sqrt{7}}{4}$$

quad III

$$\sin \theta = -\frac{\sqrt{7}}{4}$$

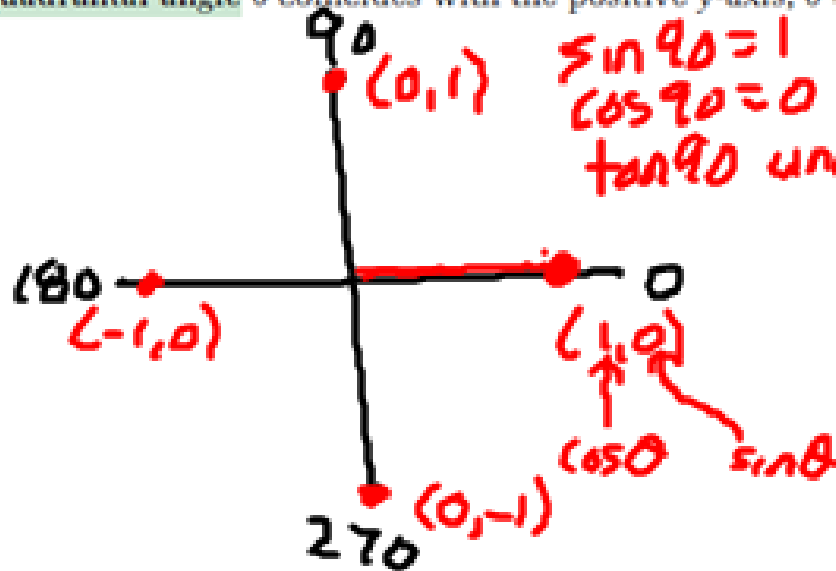
$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\tan \theta = \frac{\sqrt{7}}{3}$$

Example 4

Determine Trigonometric Ratios of Quadrantal Angles

Determine the values of $\sin \theta$, $\cos \theta$, and $\tan \theta$ when the terminal arm of **quadrantal angle** θ coincides with the positive y-axis, $\theta = 90^\circ$.



quadrantal angle

- an angle in standard position whose terminal arm lies on one of the axes
- examples are 0° , 90° , 180° , 270° , and 360°

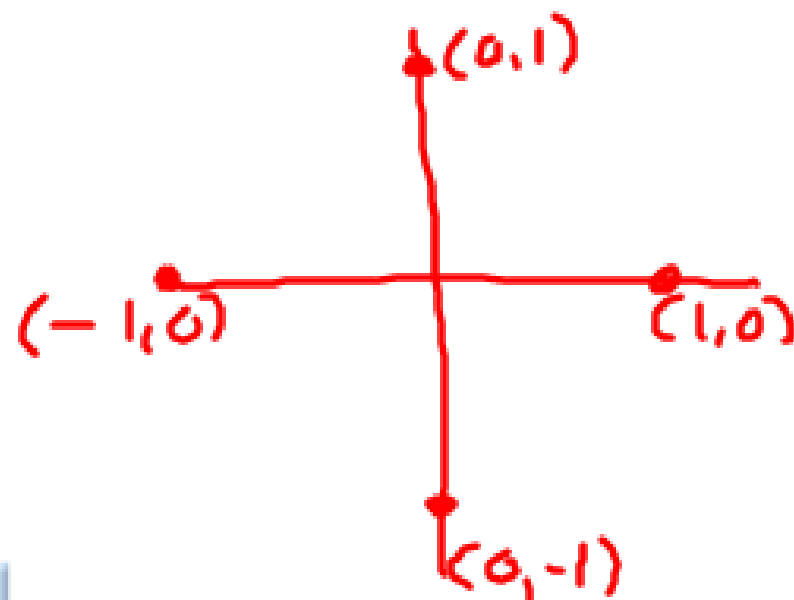
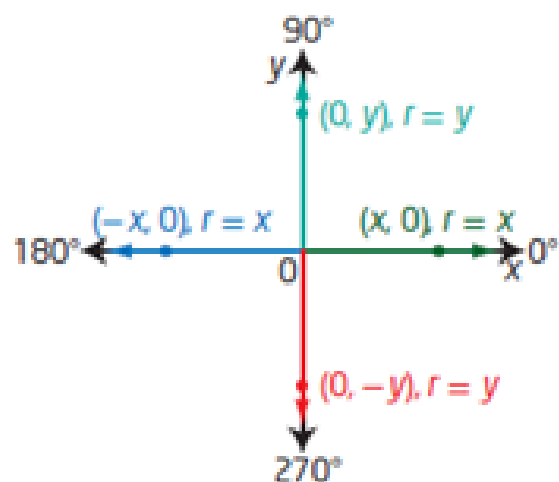
$$\begin{aligned} \cos 0 &= 1 \\ \sin 0 &= 0 \\ \tan 0 &= 0 \end{aligned} \quad \left(\frac{\sin 0}{\cos 0} \right)$$



$$\begin{aligned} \cos \theta &= \frac{x}{r} & \sin \theta &= \frac{y}{r} \\ \cos &= x \end{aligned}$$

Your Turn

Use the diagram to determine the values of $\sin \theta$, $\cos \theta$, and $\tan \theta$ for quadrantal angles of 0° , 180° , and 270° . Organize your answers in a table as shown below.



	0°	90°	180°	270°
$\sin \theta$	0	1	0	-1
$\cos \theta$	1	0	-1	0
$\tan \theta$	0	undefined	0	undefined.

$$(\cos \theta, \sin \theta)$$

$$\tan = \frac{\sin}{\cos}$$

Example 5

Solve for an Angle Given Its Exact Sine, Cosine, or Tangent Value

Solve for θ .

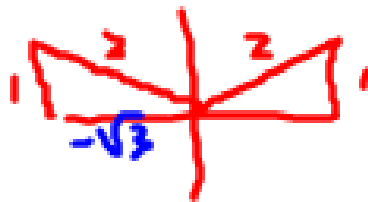
a) $\sin \theta = 0.5, 0^\circ \leq \theta < 360^\circ$

b) $\cos \theta = -\frac{\sqrt{3}}{2}, 0^\circ \leq \theta < 180^\circ$

$$\sin \theta = \sin(180 - \theta)$$

a) calculator says 30° , 150°

$$\cos \theta = \cos(360 - \theta)$$



b) $\theta = 150^\circ$
(or 210°)

Example 6

Solve for an Angle Given Its Approximate Sine, Cosine, or Tangent Value

Given $\cos \theta = -0.6753$, where $0^\circ \leq \theta < 360^\circ$, determine the measure of θ , to the nearest tenth of a degree.

calculator (degrees)

$$\cos^{-1}(-0.6753)$$

$$\theta = 132.5^\circ$$

$$\text{or } 227.5^\circ \leftarrow 360 - \text{ANS}$$

Today: 1-11 pgs.96-97

Monday: 12-22 pgs.97-98