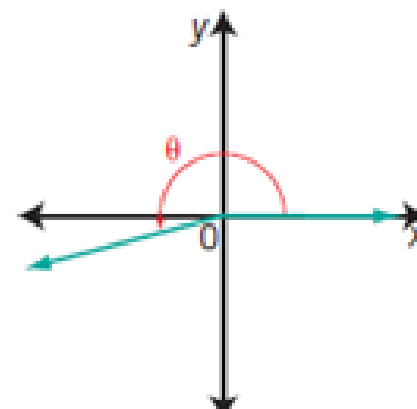
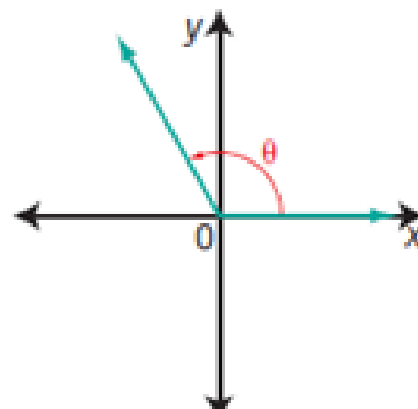
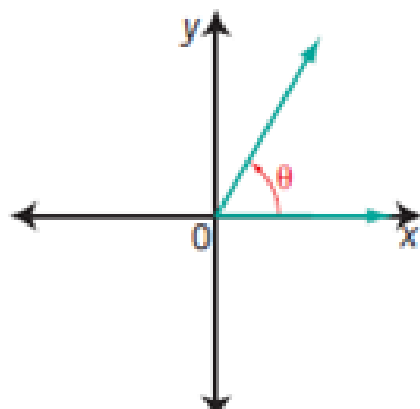
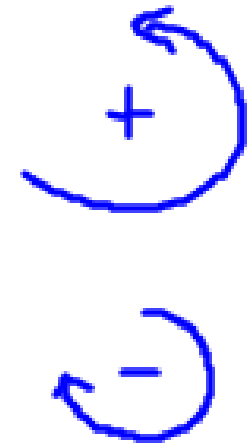
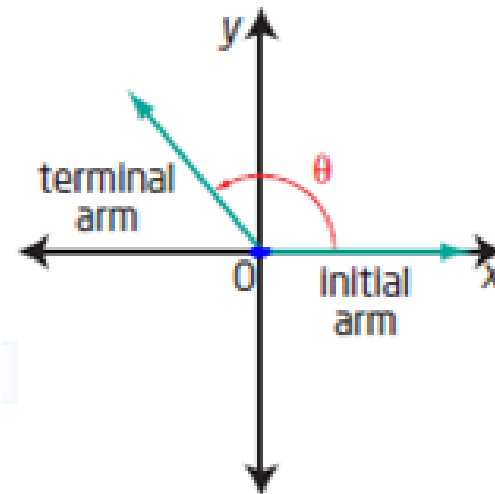
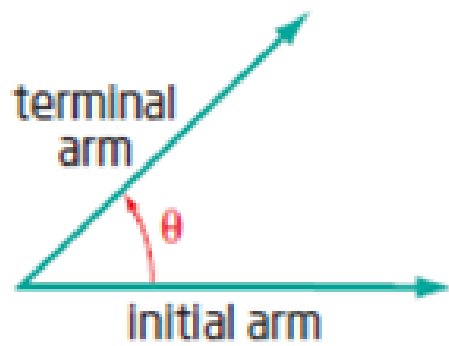
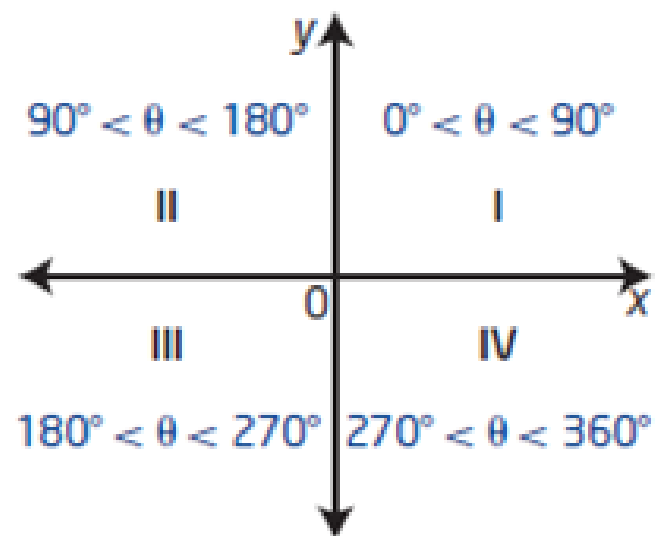
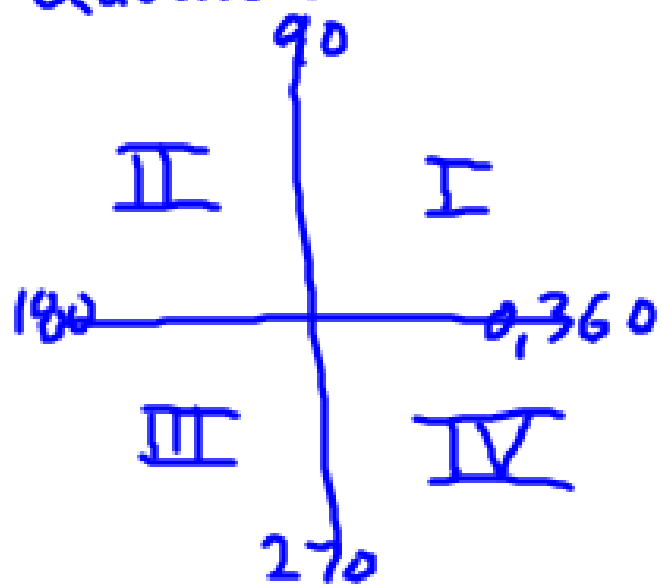
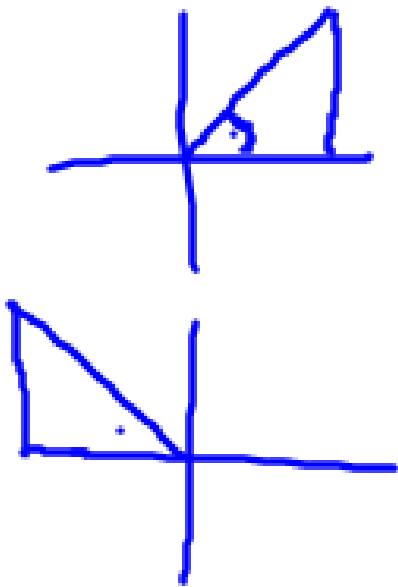


2.1 Angles in Standard Position



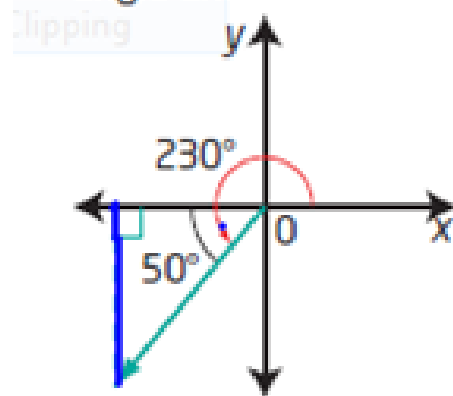
Quadrants



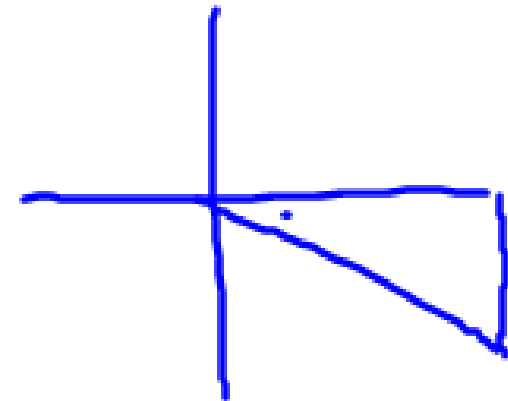


reference angle

- the acute angle whose vertex is the origin and whose arms are the terminal arm of the angle and the x-axis



- the reference angle for 230° is 50°

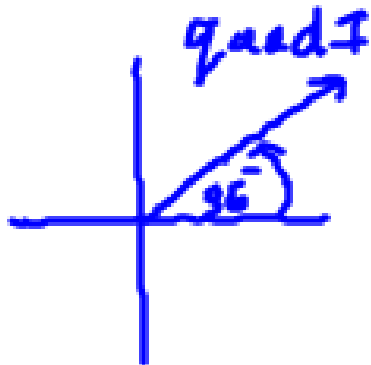


Example 1

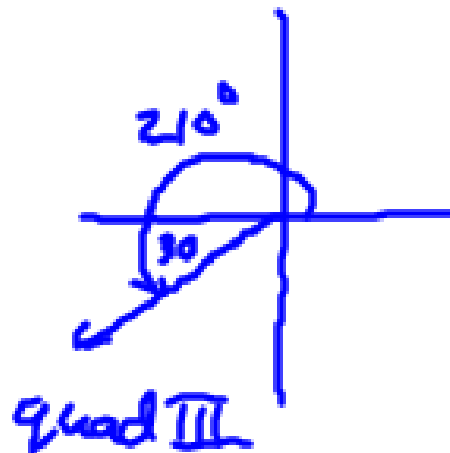
Sketch an Angle in Standard Position, $0^\circ \leq \theta < 360^\circ$

Sketch each angle in standard position. State the quadrant in which the terminal arm lies.

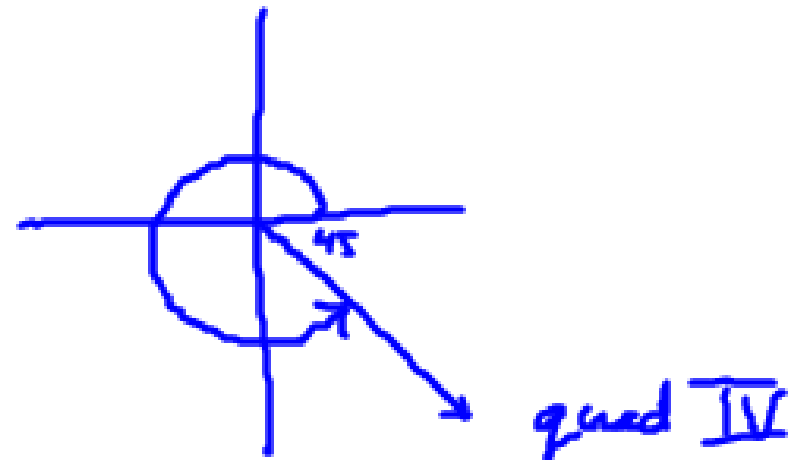
a) 36°



b) 210°



c) 315°

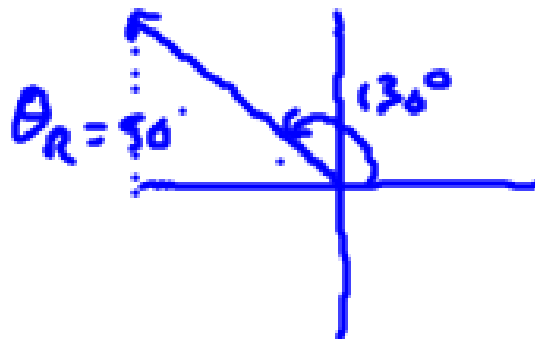


Example 2

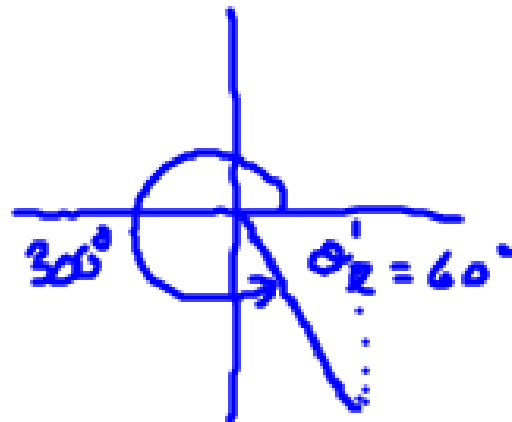
Determine a Reference Angle

Determine the reference angle θ_R for each angle θ . Sketch θ in standard position and label the reference angle θ_R .

a) $\theta = 130^\circ$



b) $\theta = 300^\circ$

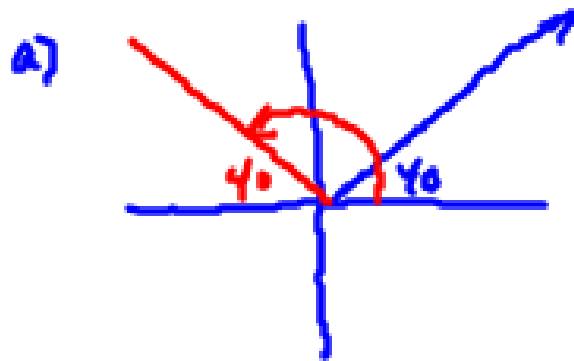


Example 3

Determine the Angle in Standard Position

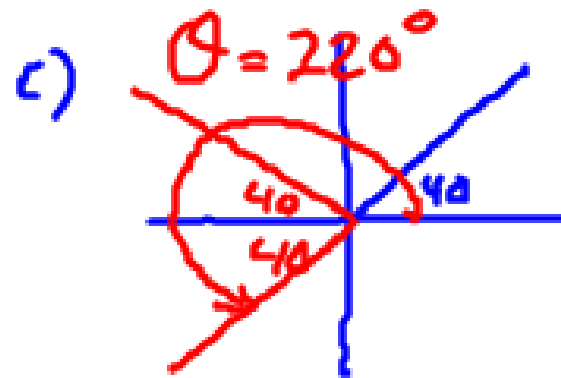
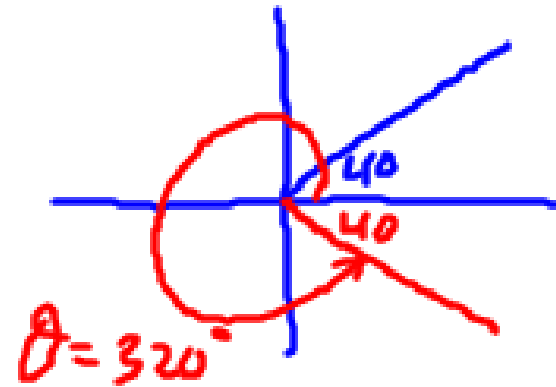
Determine the angle in standard position when an angle of 40° is reflected

- in the y -axis
- in the x -axis
- in the y -axis and then in the x -axis



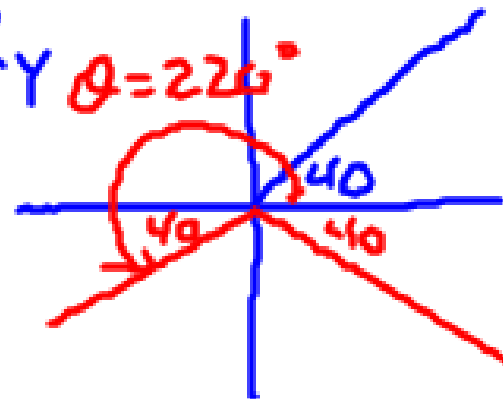
$$\theta = 140^\circ$$

b)

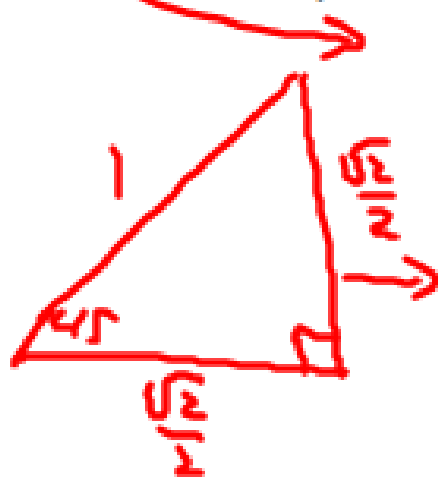
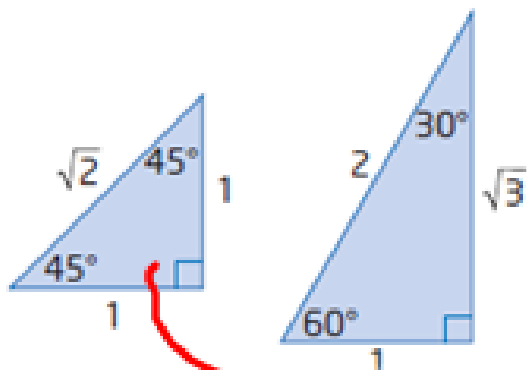


R_y, R_x

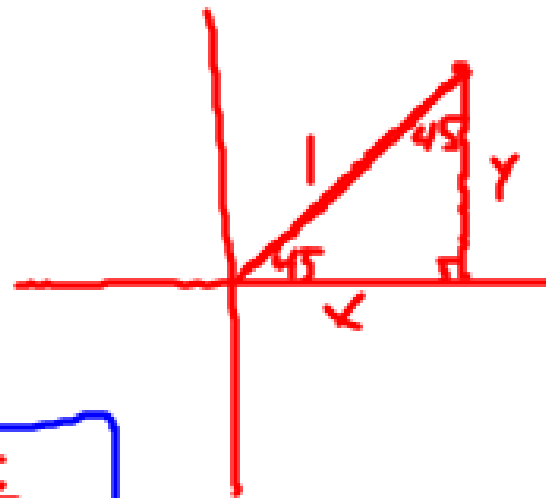
d) R_x, R_y $\theta = 220^\circ$



- You can determine exact trigonometric ratios for angles of 30° , 45° , and 60° using special triangles.

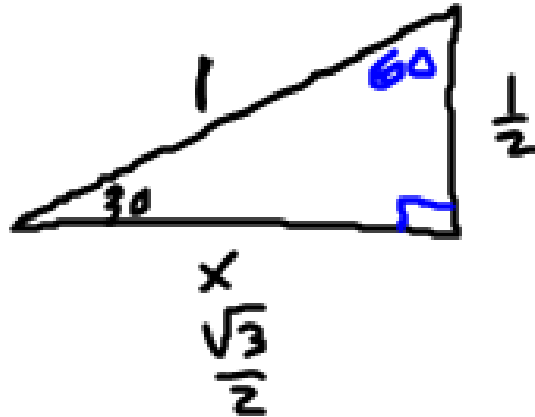
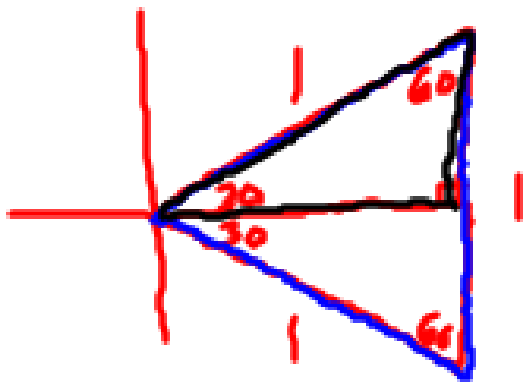


$$\begin{aligned} \sin 45^\circ &= \frac{\sqrt{2}}{2} \\ \cos 45^\circ &= \frac{\sqrt{2}}{2} \\ \tan 45^\circ &= 1 \end{aligned}$$



$$x = \frac{\sqrt{2}}{2}$$

$$\begin{aligned} x^2 + y^2 &= 1^2 \\ \text{but } x &= y \\ x^2 + x^2 &= 1 \\ 2x^2 &= 1 \\ x^2 &= \frac{1}{2} \\ x &= \sqrt{\frac{1}{2}} \\ x &= \frac{\sqrt{2}}{2} \\ x &= \frac{\sqrt{2}}{2} \end{aligned}$$



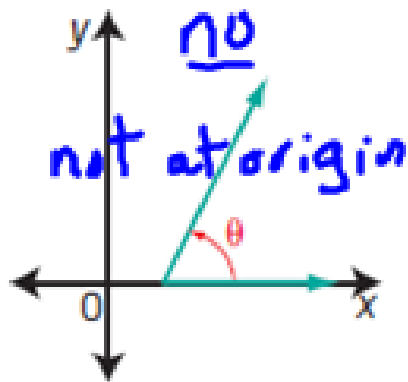
$$\begin{aligned} \sin 30^\circ &= \frac{1}{2} & \sin 60^\circ &= \frac{\sqrt{3}}{2} \\ \cos 30^\circ &= \frac{\sqrt{3}}{2} & \cos 60^\circ &= \frac{1}{2} \\ \tan 30^\circ &= \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} & \tan 60^\circ &= \sqrt{3} \end{aligned}$$

$$\begin{aligned} x^2 + \left(\frac{1}{2}\right)^2 &= 1^2 \\ x^2 + \frac{1}{4} &= 1 \\ x^2 &= 1 - \frac{1}{4} \\ x &= \sqrt{\frac{3}{4}} \\ x &= \frac{\sqrt{3}}{2} \end{aligned}$$

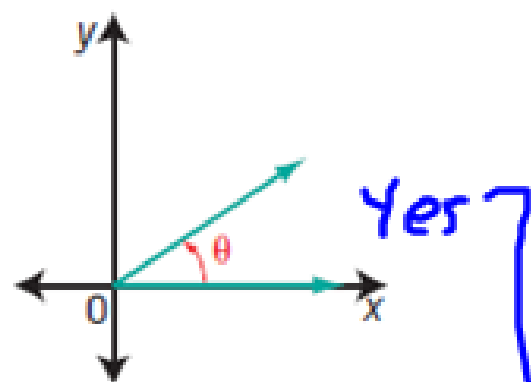
Practise

1. Is each angle, θ , in standard position?
Explain.

a)

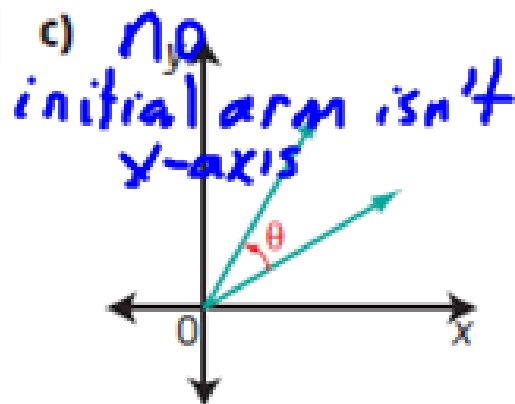


b)

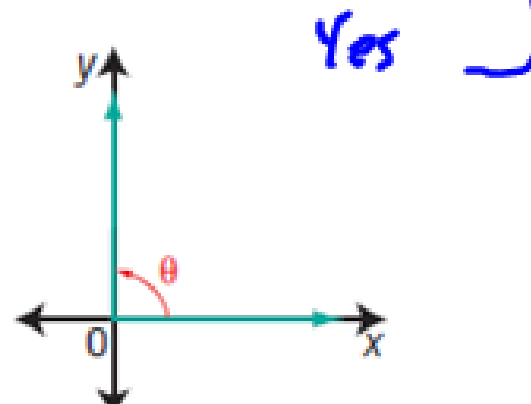


initial arm x axis
rotating about origin.

c)

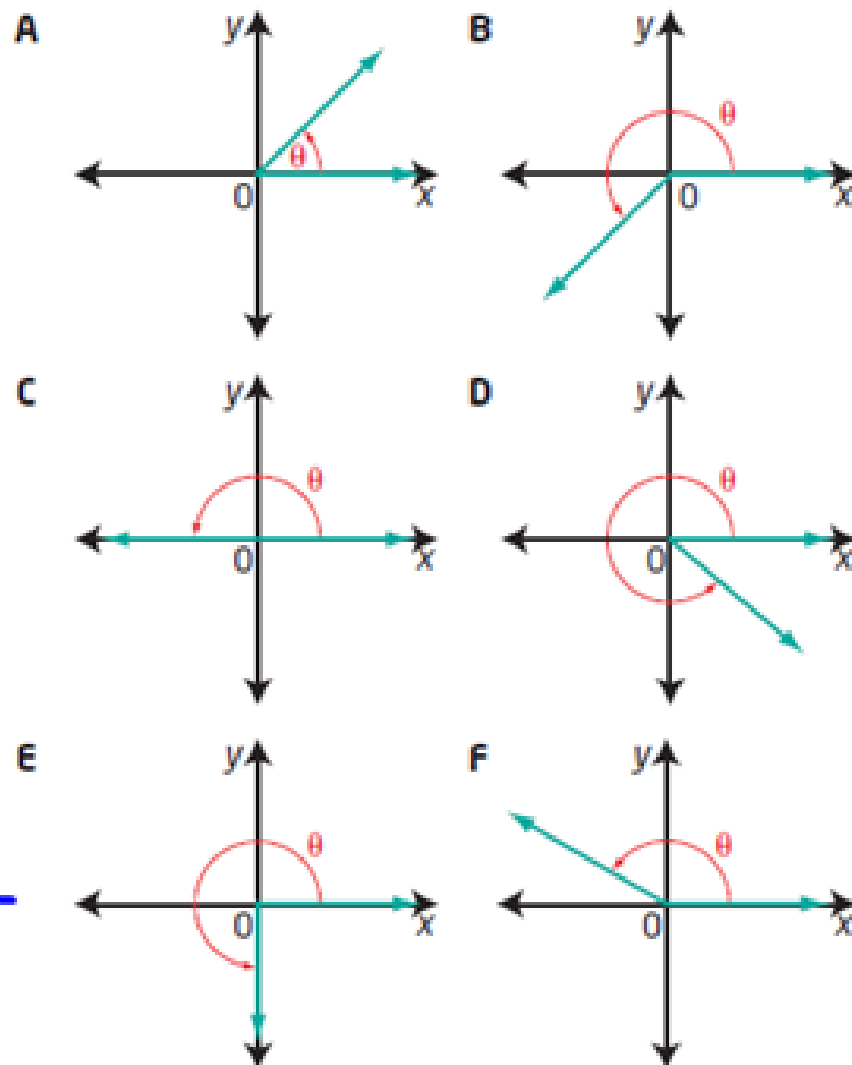
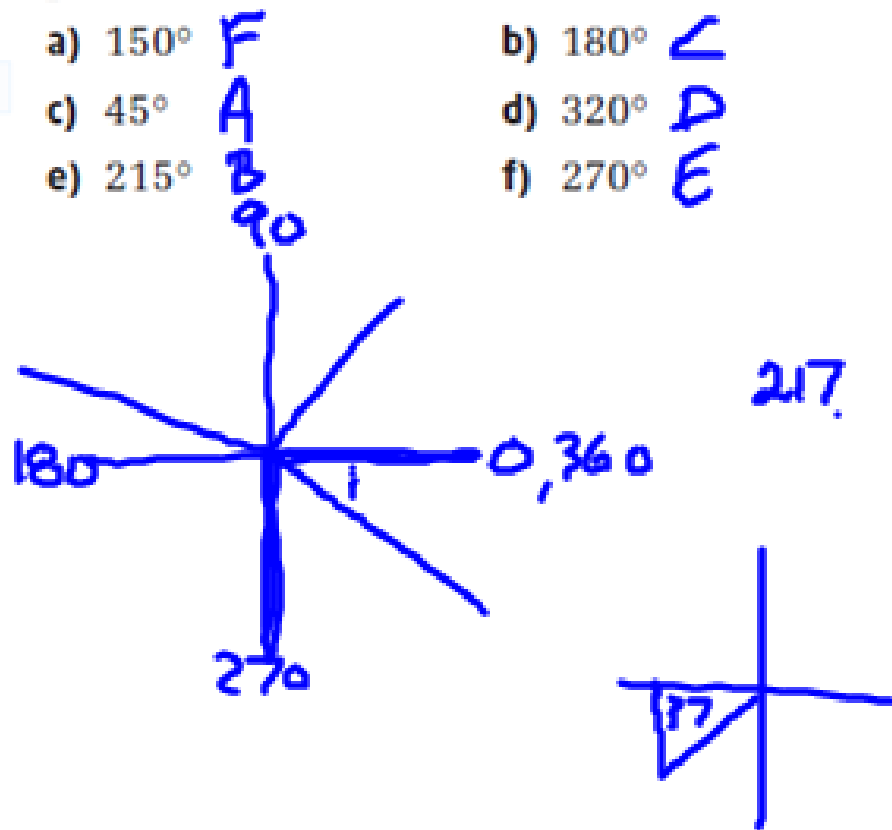


d)



2. Without measuring, match each angle with a diagram of the angle in standard position.

- | | | | |
|----------------|---|----------------|---|
| a) 150° | F | b) 180° | C |
| c) 45° | A | d) 320° | D |
| e) 215° | B | f) 270° | E |



3. In which quadrant does the terminal arm of each angle in standard position lie?

a) 48° I

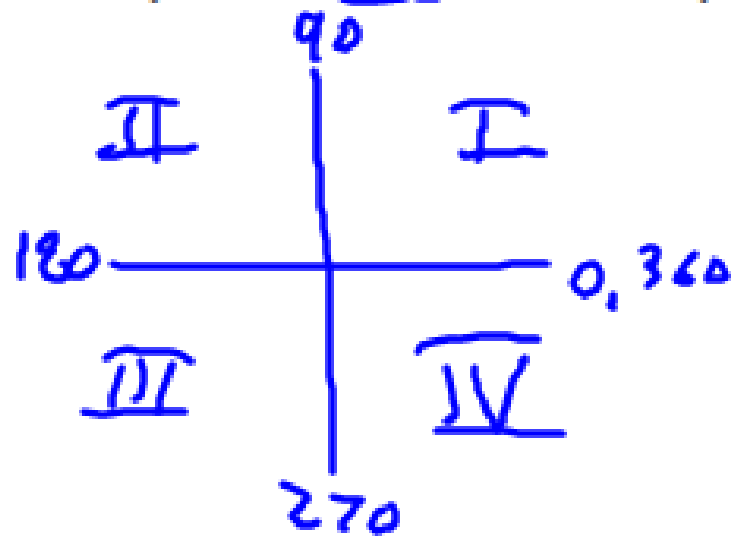
b) 300° IV

c) 185° III

d) 75° I

e) 220° III

f) 160° II



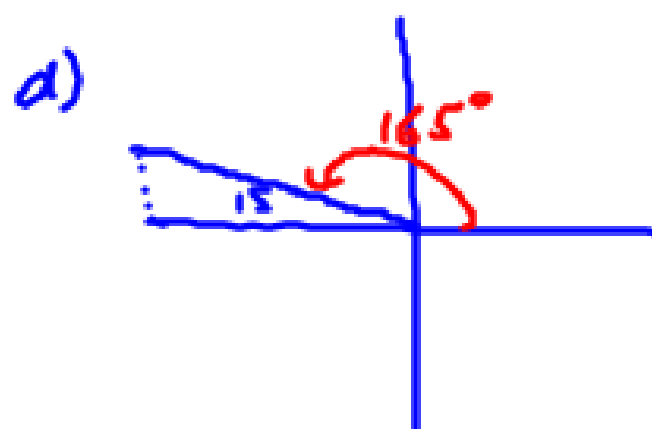
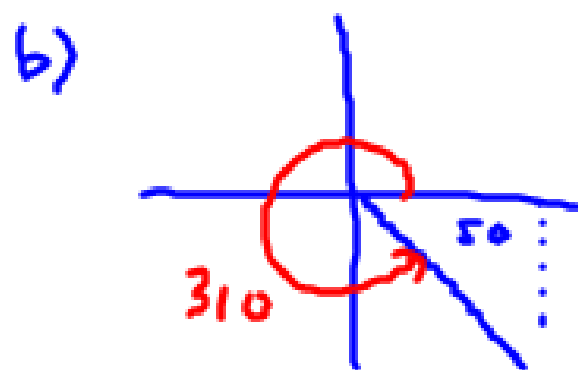
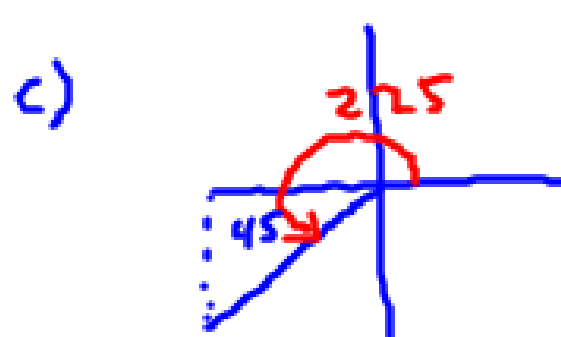
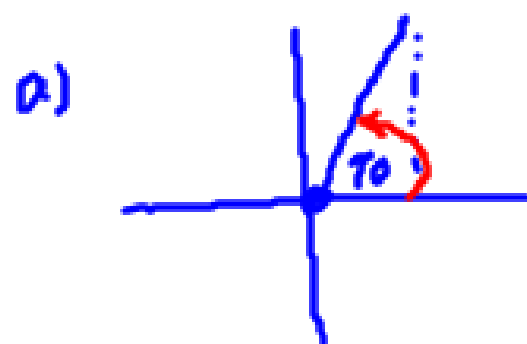
4. Sketch an angle in standard position with each given measure.

a) 70°

b) 310°

c) 225°

d) 165°



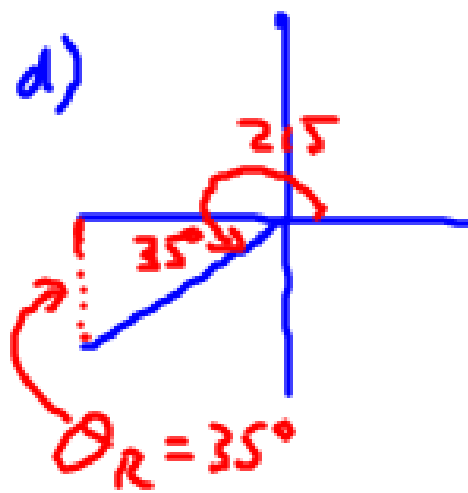
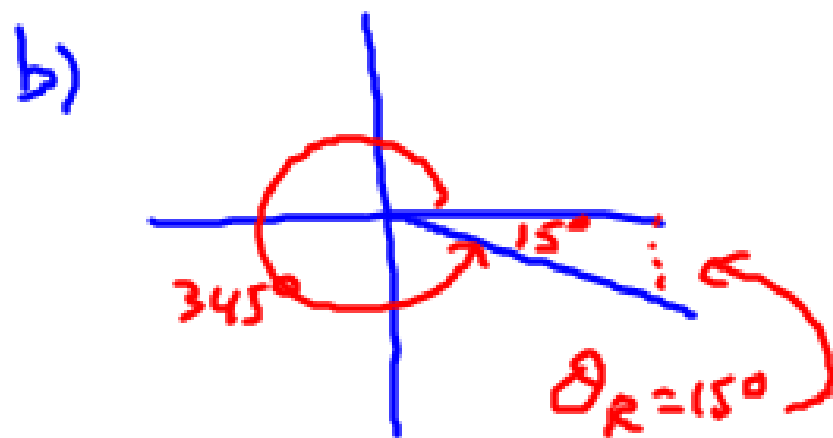
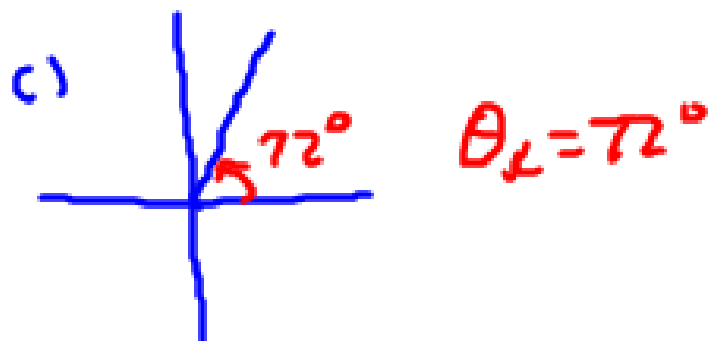
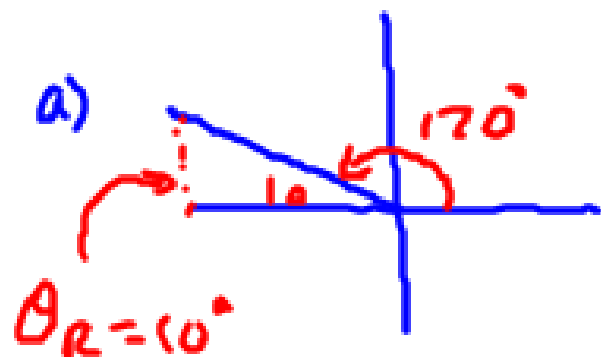
5. What is the reference angle for each angle in standard position?

a) 170°

b) 345°

c) 72°

d) 215°



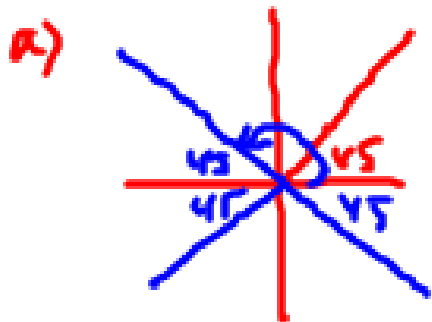
6. Determine the measure of the three other angles in standard position, $0^\circ < \theta < 360^\circ$, that have a reference angle of

a) 45°

b) 60°

c) 30°

d) 75°



$$45^\circ: 135^\circ, 225^\circ, 315^\circ$$

$$\begin{array}{ccc} 180-\theta & 180+\theta & 360-\theta \\ \downarrow & \downarrow & \downarrow \end{array}$$

$$b) 60^\circ: 120^\circ, 240^\circ, 300^\circ$$

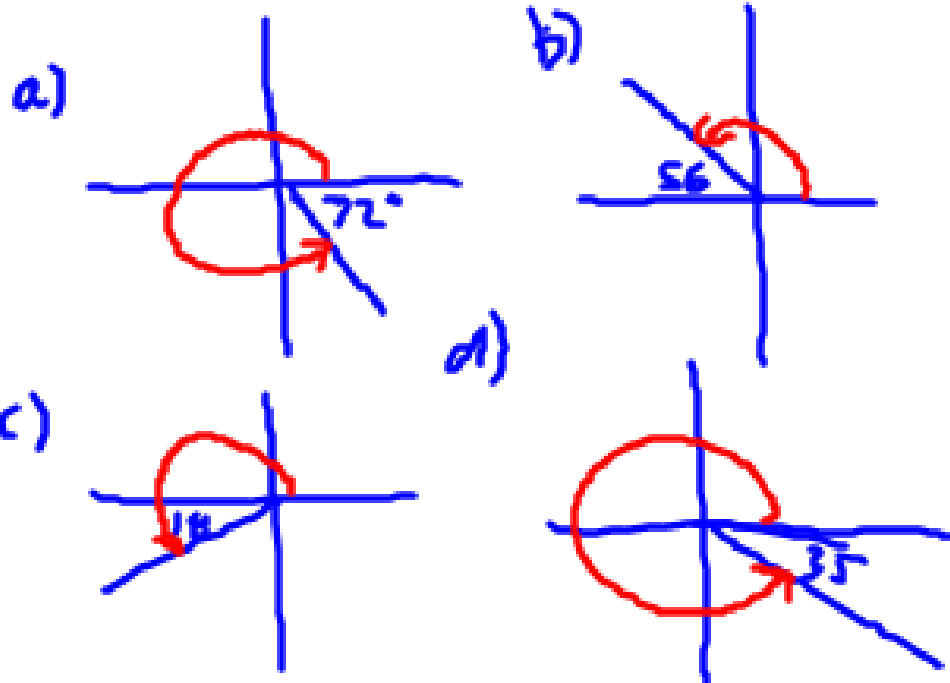
$$c) 30^\circ: 150^\circ, 210^\circ, 330^\circ$$

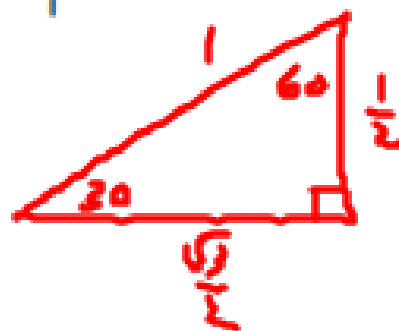
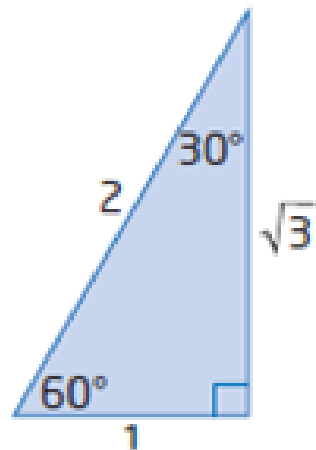
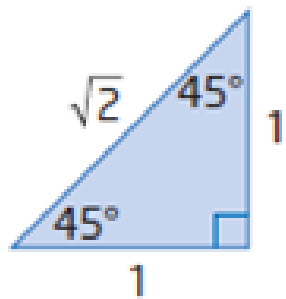
$$d) 75^\circ, 105^\circ, 255^\circ, 285^\circ$$

$$1^\circ, 179^\circ, 181^\circ, 359^\circ$$

7. Copy and complete the table. Determine the measure of each angle in standard position given its reference angle and the quadrant in which the terminal arm lies.

	Reference Angle	Quadrant	Angle in Standard Position
a)	72°	IV	288°
b)	56°	II	124°
c)	18°	III	198°
d)	35°	IV	325°



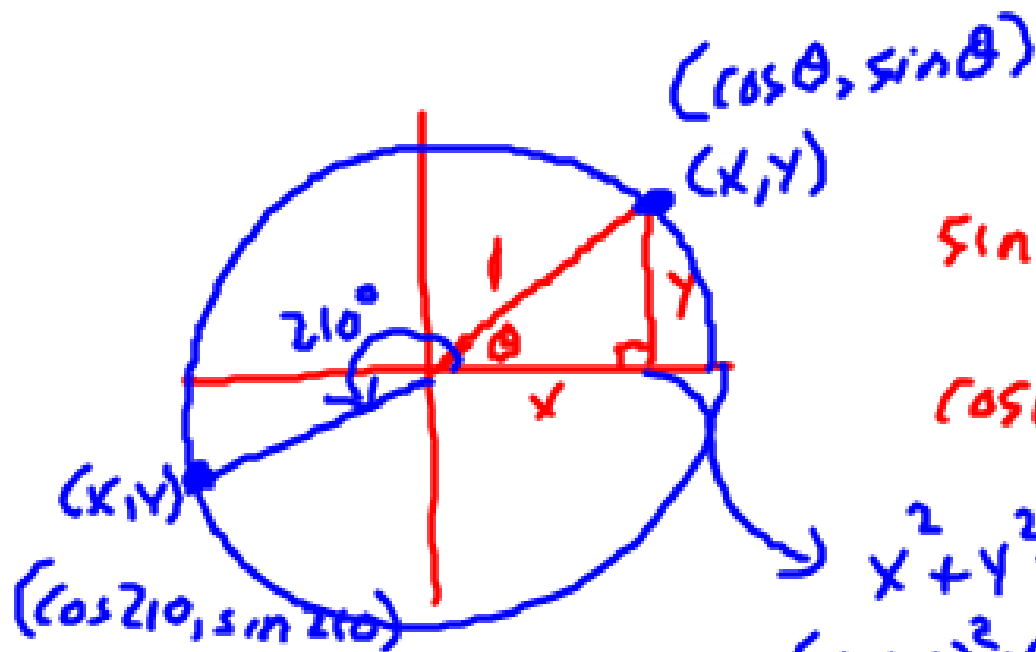


8. Copy and complete the table without using a calculator. Express each ratio using exact values.

θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
30°	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
45°	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60°	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$

$$\begin{aligned} \sin 30^\circ &= \frac{1}{2} \\ \cos 30^\circ &= \frac{\sqrt{3}}{2} \\ \tan 30^\circ &= \frac{1}{\sqrt{3}} \end{aligned}$$

$$\begin{aligned} \sin^2 \theta + \cos^2 \theta &= 1 \\ \tan &= \frac{\sin}{\cos} \end{aligned}$$



$$\sin \theta = \frac{y}{1}$$

$$y = \sin \theta$$

$$\cos \theta = \frac{x}{1}$$

$$x = \cos \theta$$

$$x^2 + y^2 = 1^2$$

$$(\cos \theta)^2 + (\sin \theta)^2 = 1$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan \theta = \frac{y}{x}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

Do: 9, 10, 11, 14, 16, 17 pgs. 83-85