9. On the winter solstice, December 21 or 22, the power, P, in watts, received from the sun on each square metre of Earth can be determined using the equation $P = 1000 (\sin x \cos 113.5° + \cos x \sin 113.5°)$, where x is the latitude of the location in the northern hemisphere.

a) Use an identity to write the equation in a more useful form.

$\sin A \cos B + \cos A \sin B = \sin A+B$

$P = 1000 \sin(x+113.5)$

10. Simplify $\cos(\pi + x) + \cos(\pi - x)$.

$\cos\pi\cos x - \sin\pi\sin x + \cos\pi\cos x + \sin\pi\sin x$

$-1\cos x - 0\sin x \qquad -1\cos x + 0\sin x$

$\boxed{-2\cos x}$

14. If $(\sin x + \cos x)^2 = k$, then what is the value of $\sin 2x$ in terms of k?

$(\sin x + \cos x)^2 = k$

$\sin^2 x + 2\sin x\cos x + \cos^2 x = k$

$1 + 2\sin x\cos x = k$

$1 + \sin 2x = k$

$\sin 2x = k - 1$

$\sin 2x$

$a^2 - b^2 = (a+b)(a-b)$

15. Show that each expression can be simplified to $\cos 2x$.

a) $\cos^4 x - \sin^4 x$

b) $\dfrac{\csc^2 x - 2}{\csc^2 x}$

a) $\cos^4 x - \sin^4 x = \cos 2x$

$(\cos^2 x)^2 - (\sin^2 x)^2 =$

$(\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) =$

$1 \cdot \cos 2x =$

$\cos 2x = \cos 2x$ ✓

b) $\dfrac{\csc^2 x - 2}{\csc^2 x} = \cos 2x$

$\dfrac{\csc^2 x}{\csc^2 x} - \dfrac{2}{\csc^2 x} =$

$1 - 2\sin^2 x =$

$\cos 2x =$ ✓

19. **a)** If $\cos\theta = \frac{3}{5}$ and $0 < \theta < 2\pi$, determine the value(s) of $\sin\left(\theta + \frac{\pi}{6}\right)$.

b) If $\sin\theta = -\frac{2}{3}$ and $\frac{3\pi}{2} < \theta < 2\pi$, determine the value(s) of $\cos\left(\theta + \frac{\pi}{3}\right)$.

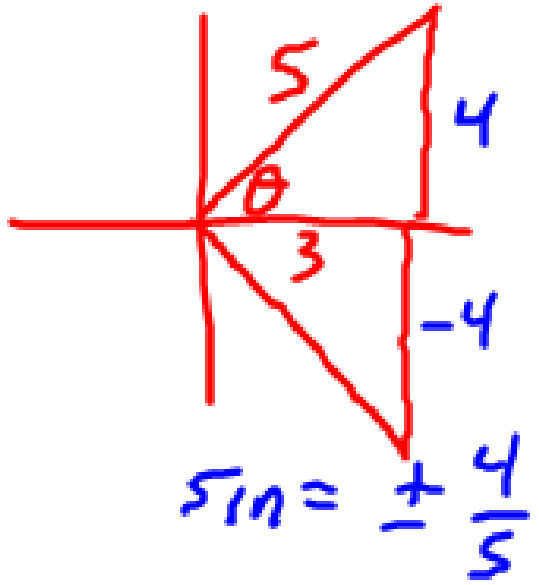
a) $\sin(\theta + \pi/6)$

$\sin\theta\cos\pi/6 + \cos\theta\sin\pi/6$

$\pm\frac{4}{5}\ \frac{\sqrt{3}}{2} + \frac{3}{5}\ \frac{1}{2}$

$\frac{3}{10} \pm \frac{4\sqrt{3}}{10}$

$\frac{3 \pm 4\sqrt{3}}{10}$



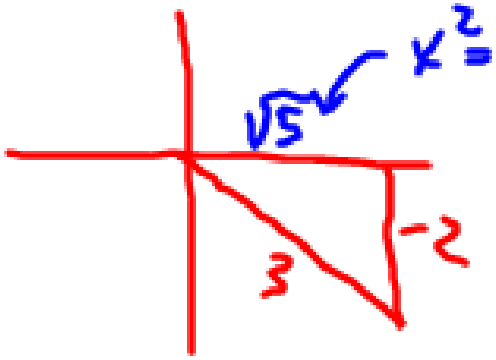
$\sin = \pm\frac{4}{5}$

b) $\cos(\theta + \pi/3)$

$\cos\theta\cos\pi/3 - \sin\theta\sin\pi/3$

$\frac{\sqrt{5}}{3}\ \frac{1}{2} + \frac{2}{3}\ \frac{\sqrt{3}}{2}$

$\frac{\sqrt{5} + 2\sqrt{3}}{6}$



$x^2 = 3^2 - (-2)^2$

$x^2 = 5$

$x = \pm\sqrt{5}$

$x = \sqrt{5}$ quad 4

$\cos\theta = \sqrt{5}/3$

8b) $\tan 165^\circ = \tan(120^\circ + 45^\circ)$

$$= \frac{\tan 120 + \tan 45}{1 - \tan 120 \tan 45}$$

$$= \frac{-\sqrt{3} + 1}{1 - -\sqrt{3}}$$

$$= \frac{1-\sqrt{3}}{1+\sqrt{3}} \left(\frac{1-\sqrt{3}}{1-\sqrt{3}}\right)$$

$$= \frac{1 - 2\sqrt{3} + 3}{1 - 3}$$

$$= \frac{4 - 2\sqrt{3}}{-2}$$

$$= \boxed{-2 + \sqrt{3}}$$

$\tan 45 = \frac{\sin 45}{\cos 45}$

$= \frac{\sqrt{2}/2}{\sqrt{2}/2}$

$= 1$

$\tan 120 = \frac{\sin 120}{\cos 120}$

$= \frac{\sqrt{3}/2}{-1/2}$

$= -\sqrt{3}$

8e) $\csc \frac{\pi}{12}$

$\csc 15^\circ$

$\frac{\pi}{12} \cdot \frac{180}{\pi} = 15^\circ$

$\sin 15^\circ = \sin(45^\circ - 30^\circ)$

$= \sin 45 \cos 30 - \cos 45 \sin 30$

$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

$\csc 15$

$\frac{4}{\sqrt{6}-\sqrt{2}} \left(\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)$

$\frac{4\sqrt{6}+4\sqrt{2}}{6-2}$

$\frac{4\sqrt{6}+4\sqrt{2}}{4}$

$\sqrt{6}+\sqrt{2}$

6.3 Proving Identities

$\sin^2 + \cos^2 = 1$

$\tan x = \frac{\sin x}{\cos x}$

$\cot x = \frac{\cos x}{\sin x}$

Example 1

Verify Versus Prove That an Equation Is an Identity

a) Verify that $1 - \sin^2 x = \sin x \cos x \cot x$ for some values of x. Determine the non-permissible values for x. Work in degrees.

b) Prove that $1 - \sin^2 x = \sin x \cos x \cot x$ for all permissible values of x.

$1 - \sin^2 x = \sin x \cos x \cot x$

$\cos^2 x = \sin x \cos x \frac{\cos x}{\sin x}$

$\cos^2 x = \cos^2 x$ ✓

Example 2

Prove an Identity Using Double-Angle Identities

Prove that $\tan x = \dfrac{1 - \cos 2x}{\sin 2x}$ is an identity for all permissible values of x.

$$\tan x = \frac{1 - \cos 2x}{\sin 2x}$$

$$= \frac{1 - (1 - 2\sin^2 x)}{2 \sin x \cos x}$$

$$= \frac{2 \sin^2 x}{2 \sin x \cos x}$$

$$= \frac{\sin x}{\cos x}$$

$$= \tan x \quad \checkmark$$

Example 3

Prove More Complicated Identities

Prove that $\frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$ is an identity for all permissible values of x.

$$\frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x}\left(\frac{1-\cos x}{1-\cos x}\right)$$

$$= \frac{\sin x\,(1-\cos x)}{1-\cos^2 x}$$

$$= \frac{\sin x\,(1-\cos x)}{\sin^2 x}$$

$$= \frac{1-\cos x}{\sin x} \checkmark$$

Example 4

Prove an Identity That Requires Factoring

Prove the identity $\cot x - \csc x = \dfrac{\cos 2x - \cos x}{\sin 2x + \sin x}$ for all permissible values of x.

$$\cot x - \csc x = \frac{\cos 2x - \cos x}{\sin 2x + \sin x}$$

$$\frac{\cos x}{\sin x} - \frac{1}{\sin x} = \frac{2\cos^2 x - 1 - \cos x}{2\sin x \cos x + \sin x}$$

← look for factoring

$$\frac{\cos x - 1}{\sin x} = \frac{2\cos^2 - \cos x - 1}{\sin x(2\cos x + 1)}$$

$2a^2 - a - 1$

$(2a+1)(a-1)$

$$= \frac{(2\cos x + 1)(\cos x - 1)}{\sin x\,(2\cos x + 1)}$$

$$= \frac{\cos x - 1}{\sin x}$$

- It is usually easier to make a complicated expression simpler than it is to make a simple expression more complicated.
- Some strategies that may help you prove identities include:
  - Use known identities to make substitutions.
  - If quadratics are present, the Pythagorean identity or one of its alternate forms can often be used.
  - Rewrite the expression using sine and cosine only.
  - Multiply the numerator and the denominator by the conjugate of an expression.
  - Factor to simplify expressions.

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