6.2 Sum/Difference/Double angle Identities

$$\left.\begin{aligned} \sin(A+B) &= \sin A\cos B + \cos A\sin B \\ \cos(A+B) &= \cos A\cos B - \sin A\sin B \\ \tan(A+B) &= \frac{\tan A + \tan B}{1 - \tan A\tan B} \end{aligned}\right] \text{sum}$$

$$\left.\begin{aligned} \sin(A-B) &= \sin A\cos B - \cos A\sin B \\ \cos(A-B) &= \cos A\cos B + \sin A\sin B \\ \tan(A-B) &= \frac{\tan A - \tan B}{1 + \tan A\tan B} \end{aligned}\right] \text{difference}$$

$$\left.\begin{aligned} \sin 2A &= 2\sin A\cos A \\ \cos 2A &= \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A \\ \tan 2A &= \frac{2\tan A}{1-\tan^2 A} \end{aligned}\right] \text{double}$$

ex1 pg 301 a) $\sin 48\cos 17 - \cos 48\sin 17$

$$* \left[\begin{array}{c} \sin A\cos B - \cos A\sin B \\ \sin(A-B) \end{array}\right]$$

$$\sin(48-17)$$

$$\sin 31^\circ$$

* table pg 305 (will be given)

b) $\cos^2 \pi/3 - \sin^2 \pi/3$

$$* \left[\begin{array}{c} \cos^2 A - \sin^2 A \\ \cos 2A \end{array}\right]$$

$$\cos 2\pi/3 \quad \text{OR} \quad -1/2$$

ex2 book gives $\cos 2A = \cos^2 A - \sin^2 A$

$$= \cos^2 A - (1 - \cos^2 A) \quad \text{Pythagorean Identity}$$

$$= 2\cos^2 A - 1$$

Do "Your turn" pg 301, 302

ex3 $\dfrac{1-\cos 2x}{\sin 2x} = \dfrac{1-(1-2\sin^2 x)}{2\sin x\cos x}$ ← restrictions (a) $\sin x, \cos x \neq 0$

$= \dfrac{2\sin^2 x}{2\sin x\cos x}$ $\quad x \neq \pi/2k,\ k \in I$

$= \dfrac{\sin x}{\cos x}$

(b) $= \tan x$

ex4 a) $\sin \pi/12$

$\sin 15°$

$\sin(45°-30°)$

$\sin 45 \cos 30 - \cos 45 \sin 30$

$\frac{\sqrt{2}}{2}\ \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\ \frac{1}{2}$

$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$\frac{\sqrt{6}-\sqrt{2}}{4}$

b) $\tan 105°$ $\quad \dfrac{\sin 105°}{\cos 105°} \quad \dfrac{\sin(60+45)}{\cos(60+45)}$

$\dfrac{\sin 60\cos 45 + \cos 60 \sin 45}{\cos 60 \cos 45 - \sin 60 \sin 45}$

$\dfrac{\frac{\sqrt{3}}{2}\ \frac{\sqrt{2}}{2} + \frac{1}{2}\ \frac{\sqrt{2}}{2}}{\frac{1}{2}\ \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\ \frac{\sqrt{2}}{2}}$

$\dfrac{\frac{\sqrt{6}+\sqrt{2}}{4}}{\frac{\sqrt{2}-\sqrt{6}}{4}}$

$\dfrac{\sqrt{6}+\sqrt{2}}{\sqrt{2}-\sqrt{6}}$ * book

$\dfrac{\sqrt{6}+\sqrt{2}}{\sqrt{2}-\sqrt{6}}\left(\dfrac{\sqrt{2}+\sqrt{6}}{\sqrt{2}+\sqrt{6}}\right) = \dfrac{\sqrt{12}+6+2+\sqrt{12}}{2+6}$ $\quad \dfrac{8+2\sqrt{12}}{8}$

$\dfrac{4+\sqrt{12}}{4}$ $\quad \dfrac{4+2\sqrt{3}}{4}$

* $\dfrac{2+\sqrt{3}}{2}$

pg 306 1-9a, 10, 14, 15, 19

1 a) $\cos 43 \cos 27 - \sin 43 \sin 27$ $\quad \cos(A+B)$

$\therefore \cos 70^\circ$

b) $\sin 15 \cos 20 + \cos 15 \sin 20$ $\quad \sin(A+B)$

$\therefore \sin 35^\circ$

c) $\cos^2 19 - \sin^2 19$ $\quad \cos 2A$

$\cos 38^\circ$

e) $8 \sin \pi/3 \cos \pi/3$

$4 \cdot 2 \sin \pi/3 \cos \pi/3$ $\quad 4 \cdot \sin 2A$

$4 \sin 2\pi/3$

d) $\sin \frac{3\pi}{2} \cos \frac{5\pi}{4} - \cos \frac{3\pi}{2} \sin \frac{5\pi}{4}$ $\quad \sin(A-B)$

$\sin \pi/4$ $\quad (\sqrt{2}/2)$

2 a) $\cos 40 \cos 20 - \sin 40 \sin 20$ $\quad \cos(A+B)$

$\cos 60^\circ$ $\quad (\frac{1}{2})$

b) $\sin 20 \cos 25 + \cos 20 \sin 25$ $\quad \sin(A+B)$

$\sin 45$ $\quad (\frac{\sqrt{2}}{2})$

c) $\cos^2 \pi/6 - \sin^2 \pi/6$ $\quad \cos 2A$

$\cos \pi/3$ $\quad (\frac{1}{2})$

d) $\cos \pi/2 \cos \pi/3 - \sin \pi/2 \sin \pi/3$ $\quad \cos(A+B)$

$\cos 5\pi/6$ $\quad (-\sqrt{3}/2)$

3. $1 - \cos 2x$

$1 - (1 - 2\sin^2 x)$

$2 \sin^2 x$

4. a) $2 \sin \pi/4 \cos \pi/4$ $\quad 2 \sin A \cos A \rightarrow \sin 2A$

$\sin \pi/2$ $\quad (1)$

b) $6 (\cos^2 24 - \sin^2 24) \tan 48$

$6 \cos 48^\circ \left(\frac{\sin 48}{\cos 48}\right)$ $\quad 6 \sin 48^\circ$

4 c) $\dfrac{2\tan 76}{1-\tan^2 76} \rightarrow \tan 152^\circ$

d) $2\cos^2 \pi/6 - 1 \rightarrow \cos \pi/3 \qquad (1/2)$

e) $1 - 2\cos^2 \pi/12$

$-(2\cos^2 \pi/12 - 1) \qquad -\cos \pi/6 \qquad (-\sqrt{3}/2)$

5 a) $\dfrac{\sin 2\theta}{2\cos\theta} \rightarrow \dfrac{2\sin\theta\cos\theta}{2\cos\theta} \rightarrow \sin\theta$

b) $\cos 2x \cos x + \sin 2x \sin x \rightarrow \cos(2x - x) \rightarrow \cos x$

c) $\dfrac{\cos 2\theta + 1}{2\cos\theta} \rightarrow \dfrac{(2\cos^2\theta - 1) + 1}{2\cos\theta} \rightarrow \dfrac{2\cos^2\theta}{2\cos\theta} \rightarrow \cos\theta$

d) $\dfrac{\cos^3 x}{\cos 2x + \sin^2 x} \rightarrow \dfrac{\cos^3 x}{(1 - 2\sin^2 x) + \sin^2 x} \rightarrow \dfrac{\cos^3 x}{1 - \sin^2 x} \rightarrow \dfrac{\cos^3 x}{\cos^2 x} \rightarrow \cos x$

6. $\sin(90 - 60) = \sin 90 - \sin 60$

$\sin 30^\circ = 1 - \sqrt{3}/2$

$1/2 = \dfrac{2-\sqrt{3}}{2}$ not equal.

7. $\cos(90 - x) = \cos 90 \cos x + \sin 90 \sin x$

$= 0(\cos x) + 1 \sin x$

$= \sin x$

8a) $\cos 75 = \cos(45 + 30)$

$= \cos 45 \cos 30 - \sin 45 \sin 30$

$\dfrac{\sqrt{2}}{2}\,\dfrac{\sqrt{3}}{2} - \dfrac{\sqrt{2}}{2}\,\dfrac{1}{2}$

$\dfrac{\sqrt{6} - \sqrt{2}}{4}$

8b) $\tan 165 = \dfrac{\sin 165}{\cos 165} = \dfrac{\sin(120+45)}{\cos(120+45)} \quad \dfrac{\sin 120\cos 45 + \cos 120 \sin 45}{\cos 120 \cos 45 - \sin 120 \sin 45}$

$$= \frac{\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} + \frac{-1}{2}\frac{\sqrt{2}}{2}}{-\frac{1}{2}\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2}} = \frac{\sqrt{6}-\sqrt{2}}{-\sqrt{2}-\sqrt{6}}\left(\frac{-\sqrt{2}+\sqrt{6}}{-\sqrt{2}+\sqrt{6}}\right)$$

$$\frac{6+2-2\sqrt{12}}{2-6} = \frac{8-4\sqrt{3}}{-4}$$

$$= \sqrt{3}-2$$

c) $\sin\frac{7\pi}{12} = \sin(\pi/3 + \pi/4) = \sin\pi/3\cos\pi/4 + \cos\pi/3\sin\pi/4$

$$= \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} + \frac{1}{2}\frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6}+\sqrt{2}}{4}$$

d) $\cos 195^\circ = \cos(150+45) = \cos 150\cos 45 - \sin 150 \sin 45$

$$-\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2}$$

$$= \frac{-\sqrt{6}-\sqrt{2}}{4}$$

e) $\cos\pi/12 = \cos(\pi/3 - \pi/4) = \cos\pi/3\cos\pi/4 + \sin\pi/3\sin\pi/4$

$$\frac{1}{2}\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{2}+\sqrt{6}}{4}$$

f) $\sin -\pi/12 = \sin(\pi/4 - \pi/3) = \sin\pi/4\cos\pi/3 - \cos\pi/4\sin\pi/3$

$$\frac{\sqrt{2}}{2}\frac{1}{2} - \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} = \frac{\sqrt{2}-\sqrt{6}}{4}$$

$\csc\pi/12 \rightarrow \sin\pi/12 = \sin(\pi/3 - \pi/4) = \sin\pi/3\cos\pi/4 - \cos\pi/3\sin\pi/4$

$$\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\therefore \csc = \frac{4}{\sqrt{6}-\sqrt{2}}\left(\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)$$

$$\frac{4\sqrt{6}+4\sqrt{2}}{6-2} \quad \therefore \sqrt{6}+\sqrt{2}$$

9 a) $1000(\sin x \cos 113.5 + \cos x \sin 113.5)$

$1000 \sin(x + 113.5)$

10. $\cos(\pi + x) + \cos(\pi - x)$

$\cos\pi \cos x - \sin\pi \sin x \quad + \cos\pi \cos x + \sin\pi \sin x$

$-1 \cos x \qquad\qquad -1 \cos x$

$\therefore -2\cos x$

14. $\sin^2 x + 2\sin x \cos x + \cos^2 x = k$

$\sin^2 x + \cos^2 x + 2\sin x \cos x = k$

$1 + \sin 2x = k$

$\sin 2x = k - 1$

15. a) $(\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)$

$1 \qquad \cos 2x$

$= \cos 2x$

b) $\dfrac{\csc^2 x - 2}{\csc^2 x} = \dfrac{\csc^2 x}{\csc^2 x} - \dfrac{2}{\csc^2 x} = 1 - 2\sin^2 x = \cos 2x$

19 a) $\cos\theta = 3/5 \quad \therefore \sin^2 x = 16/25 \quad \sin x = \pm 4/5$

$\sin(\theta + \pi/6) = \sin\theta \cos\pi/6 + \cos\theta \sin\pi/6$

$\pm 4/5 \quad \sqrt{3}/2 \quad + \quad 3/5 \quad 1/2$

$3/10 \pm 4\sqrt{3}/10 = \dfrac{3 \pm 4\sqrt{3}}{10}$

b) $\sin\theta = -2/3 \quad \cos^2 = 5/9 \quad \cos\theta = \sqrt{5}/3$

$\cos(\theta + \pi/3) = \cos\theta \cos\pi/3 - \sin\theta \sin\pi/3$

$\sqrt{5}/3 \quad 1/2 \quad - \quad -2/3 \quad \sqrt{3}/2$

$\dfrac{\sqrt{5} + 2\sqrt{3}}{6}$