4.4 Trig equations

* most of the questions in 4.3 were trig equations

ex 1 pg 207 a) $5\sin\theta + 2 = 1 + 3\sin\theta$

$2\sin\theta = -1$

$\sin\theta = -1/2$ $\quad \theta = 7\pi/6, 11\pi/6$

b) $3\csc x = 6$

$\csc x = 2$ $\quad \sin x = 1/2$ $\quad x = 30°, 150°$

ex 2 $\tan^2\theta - 5\tan\theta + 4 = 0$ $\quad x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$(\tan\theta - 4)(\tan\theta - 1) = 0$

$\tan\theta = 4$ $\quad \tan\theta = 1$

$1.33, 4.47$ $\quad \pi/4, 5\pi/4$

ex 3 $\sin^2 x = 1$

$\sin x = \pm 1$

$x = \pi/2, 3\pi/2$

pgs 211-213 1-14, 18, 21

1. a) $\sin\theta = \sqrt{3}/2$

2 solutions per rotation $\therefore$ 2

b) $\cos\theta = \sqrt{2}/2$

2 solutions per rotation $\therefore$ 4

c) $\tan\theta = -1$

2 solutions per rotation 1.5 rotations $\therefore$ 3

d) sec ⊕

$\therefore$ cos ⊕

2 solutions per rotation $\therefore$ 2

2 a) $\cos\theta = \frac{1}{2}$ a) $\theta = \pi/3 + 2\pi k,\ k \in I$
b) $\theta = 5\pi/3 + 2\pi k,\ k \in I$

3 a) $\cos\theta = \sqrt{3}/2$ $\theta = \pi/6, 11\pi/6$

b) $\csc\theta = {}^{x}/_{0}$
$\sin\theta = {}^{0}/_{x} = 0°$ $\theta = 0°, 180°$

c) $-\tan^2\theta = -1$
$\tan^2\theta = 1$
$\tan\theta = \pm 1$ $\theta = -135°, -45°, 45°, 135°, 225°, 315°$

d) $\sec\theta = -\sqrt{2}$
$\cos\theta = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$ $\theta = -3\pi/4, 3\pi/4, 5\pi/4$

4. a) $\tan\theta = 4.36$ $\theta = 1.35, 4.99$
b) $\cos\theta = -0.19$ $\theta = 1.76, 4.52$
c) $\sin\theta = 0.91$ $\theta = 1.14, 2.00$
d) $\cot\theta = 12.3$ $\theta = 0.08, 3.22$
$\tan\theta = {}^{1}/_{12.3}$
e) $\sec\theta = 2.77$ $\theta = 1.20, 5.08$
$\cos\theta = {}^{1}/_{2.77}$
f) $\csc\theta = -1.57$ $\theta = -.69 \rightarrow 5.59$
$\sin\theta = {}^{1}/_{-1.57}$ $\pi - -.69 \rightarrow 3.83$

5 a) $-1 = \cos\theta$ $\theta = \pi$
b) $\tan\theta = -{}^{1}/_{\sqrt{3}}$ θ $-\pi/6, 5\pi/6, 11\pi/6$

c) $\sin x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $-315°, -225°, 45°, 135°$

d) $-1 = 2\sin x$
$-1/2 = \sin x$ $-150° - 30°$

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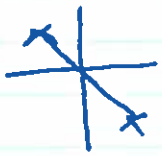
5 a) $-1 = \cot x$

$-1 = \tan x$



$x = -45°, 135°, 315°$

5 f) $\sec\theta = -\frac{2}{\sqrt{3}}$

$\cos\theta = -\frac{\sqrt{3}}{2}$

$-5\pi/6, 5\pi/6, 7\pi/6, 17\pi/6$

6

Domain	Interval
$-2\pi \le \theta \le 2\pi$	$[-2\pi, 2\pi]$
$-\pi/3 \le \theta \le 7\pi/3$	$[-\pi/3, 7\pi/3]$
$0° \le \theta \le 270°$	$[0°, 270°]$
$0° \le \theta < \pi$	$[0, \pi)$
$0° < \theta < 450°$	$(0, 450°)$
$-2\pi < \theta \le 4\pi$	$(-2\pi, 4\pi]$

7. a) $2\cos^2\theta - 3\cos\theta + 1 = 0$

$(2\cos\theta - 1)(\cos\theta - 1) = 0$

$\cos\theta = 1/2 \quad \cos\theta = 1$

$\theta = \pi/3, 5\pi/3, 0$

b) $\tan^2\theta - \tan\theta - 2 = 0$

$(\tan\theta - 2)(\tan\theta + 1) = 0$

$\tan\theta = 2 \quad \tan\theta = -1$

$63.435°, 243.435°, 135°, 315°$

c) $\sin^2\theta - \sin\theta = 0$

$\sin\theta(\sin\theta - 1) = 0$

$\sin\theta = 0 \quad \sin\theta = 1$

$\theta = 0, \pi, \pi/2$

d) $\sec^2\theta - 2\sec\theta - 3 = 0$

$(\sec\theta - 3)(\sec\theta + 1) = 0$

$\sec\theta = 3 \quad \sec\theta = -1$

$\cos\theta = 1/3 \quad \cos\theta = -1$

$\theta =$ [redacted], $-180°,$

70.529

-70.529

8. $5\cos^2\theta + 4\cos\theta = 0$

$\cos\theta(5\cos\theta + 4) = 0$

$\cos\theta = 0 \qquad \cos\theta = -4/5$

$\theta = 90°, 270° \qquad \theta = 143°, 217°$

9. $2\sin^2\theta = \sin\theta$ (a) * don't divide by $\sin\theta$ - loses a solution

$2\sin^2\theta - \sin\theta = 0$

$\sin\theta(2\sin\theta - 1) = 0$

$\sin\theta = 0 \quad \sin\theta = 1/2 \quad (0, \pi]$

$\theta = \pi \quad \pi/6, 5\pi/6$

10. $\sin\theta = 0$ on the x axis

$(\pi, 2\pi)$ doesn't touch the x-axis

11. $\sin\theta = 2 \rightarrow$ no solution $\quad -1 \le \sin\theta \le 1$

12. $\cos\theta = 1/2 \quad \theta = \begin{cases} \pi/3 + 2\pi k, k \in I \\ 5\pi/3 + 2\pi k, k \in I \end{cases}$ $\therefore$ infinite solutions.

13 a) $3\sin^2\theta - 2\sin\theta = 0$

$\sin\theta(3\sin\theta - 2) = 0$

$\sin\theta = 0 \quad \sin\theta = 2/3$

$\theta = 0, \pi \quad \theta = 0.73, 2.41$

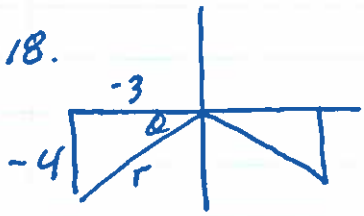
14. $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$1.00029 \sin 35 = 1.33 \sin\theta_2$

$.431 = \sin\theta_2$

$25.56° = \theta_2$ (angle of refraction)

18.



$\cot\theta = 3/4 \quad r^2 = (-3)^2 + (-4)^2$

$\tan\theta = 4/3 \quad r = 5$

$\cos\theta = -3/5$

$\sec\theta = -5/3$

21. $\cos\theta = \sqrt{3}/2$ → but θ is $x - \pi/2$

$\theta = \begin{cases} \pi/6 + 2\pi k, k \in I \\ 11\pi/6 \end{cases} \qquad x - \pi/2 = \begin{cases} \pi/6 + 2\pi k, k \in I \\ 11\pi/6 \end{cases}$

$x = \begin{cases} 2\pi/3 + 2\pi k \\ \pi/3 \end{cases}$

$\pi/3, 2\pi/3$