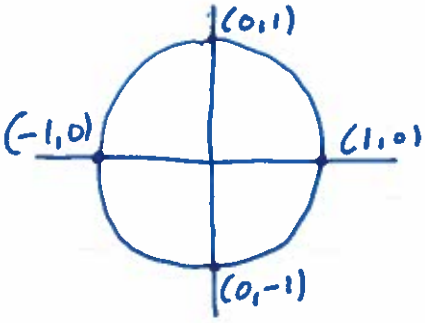
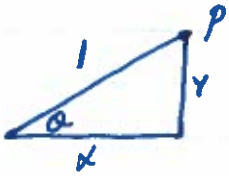
4.2 The Unit Circle



90° angles



$\sin\theta = y$

$\cos\theta = x$

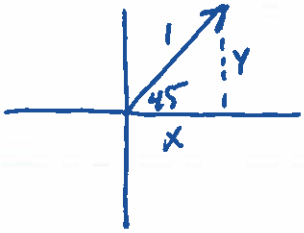
$* \quad P(x,y) = P(\cos\theta, \sin\theta)$

$\sin 0° = 0 \quad \cos 0° = 1$

$\sin 90° = 1 \quad \cos 90° = 0$

$\sin 180° = 0 \quad \cos 180° = -1$

$\sin 270° = -1 \quad \cos 270° = 0$



isosceles triangle $\therefore \quad x = y$

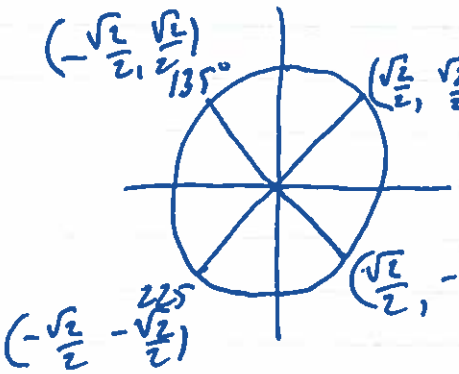
$x^2 + y^2 = 1$

$x^2 + x^2 = 1$

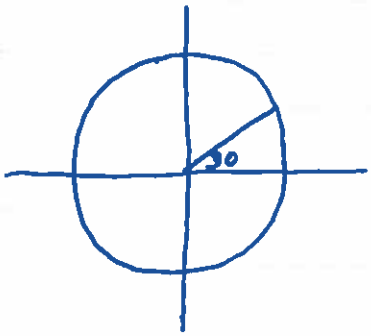
$2x^2 = 1$

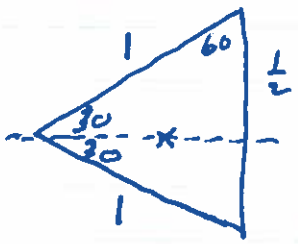
$x^2 = \frac{1}{2} \qquad x = \frac{\sqrt{2}}{2}$

45° angles



$(\cos\theta, \sin\theta)$





$x^2 + \left(\frac{1}{2}\right)^2 = 1^2$

$x^2 = \frac{3}{4}$

$x = \frac{\sqrt{3}}{2}$

$30° \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$60° \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$120° \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

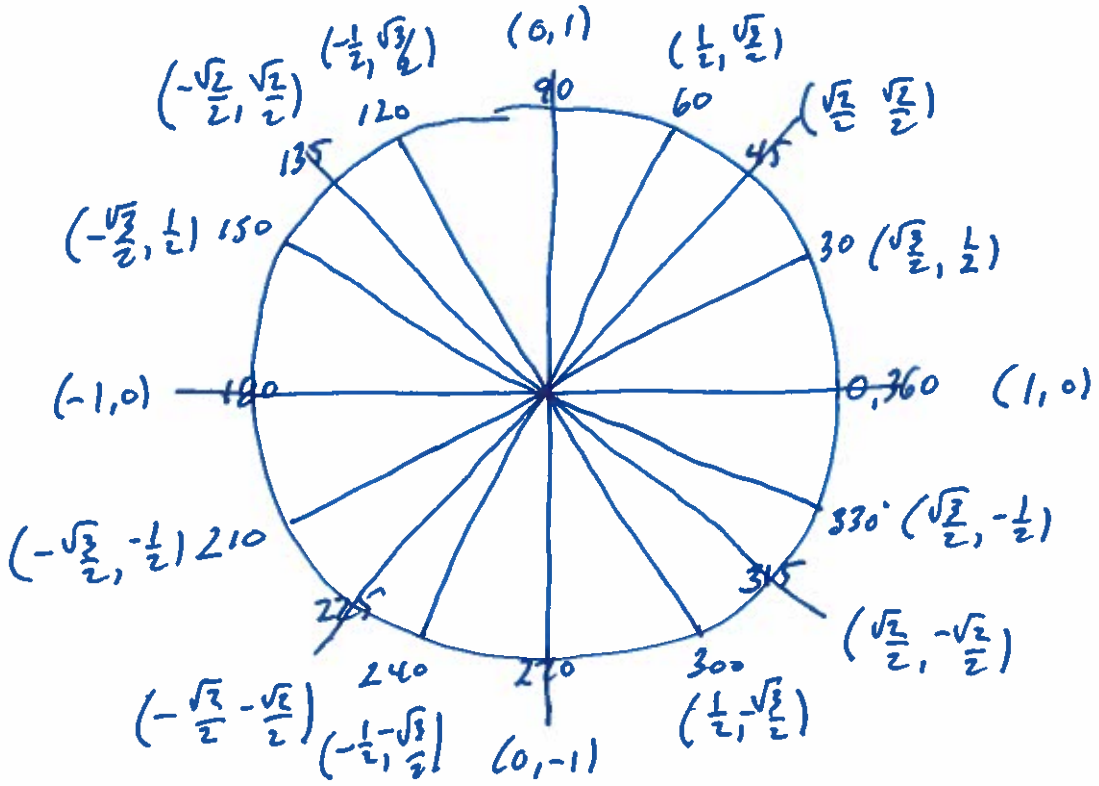
$150° \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

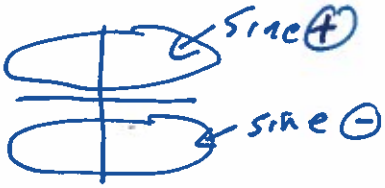
$210° \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$240° \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

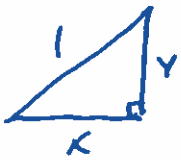
$300° \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$330° \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$





radians $\Leftrightarrow$ degrees



$x^2 + y^2 = 1 \quad \therefore \quad \cos^2\theta + \sin^2\theta = 1$

$$\sin^2\theta + \cos^2\theta = 1$$

Exercises 1-17 pgs 186-189 (skip 11, 14)

1 a) $x^2+y^2=r^2$ $x^2+y^2=16$

b) $x^2+y^2=9$ c) $x^2+y^2=144$ d) $x^2+y^2=6.76$

2. a) $(-3/4)^2+(1/4)^2 = 10/16$ no

b) $5/64+49/64 = 54/64$ no

c) $25/169+144/169 = 1$ yes

d) $16/25+9/25 = 1$ yes

e) $3/4+1/4 = 1$ yes

f) $7/16+9/16 = 1$ yes

3. a) $(1/4)^2+y^2=1$

$y^2=15/16$ $y=\pm\sqrt{15/16}=\pm\frac{\sqrt{15}}{4}$ quad I $\frac{\sqrt{15}}{4}$

b) $x^2+(\frac{2}{3})^2=1$ $x^2=5/9$ $x=\pm\sqrt{\frac{5}{9}}=\pm\frac{\sqrt{5}}{3}$

quad II $\therefore x=-\frac{\sqrt{5}}{3}$

c) $(-7/8)^2+y^2=1$ $y^2=15/64$ $y=\pm\sqrt{\frac{15}{64}}=\pm\frac{\sqrt{15}}{8}$

quad III $\therefore y=-\frac{\sqrt{15}}{8}$

d) $x^2+(-5/7)^2=1$ $x^2+25/49=1$ $x^2=24/49$ $x=\pm\sqrt{24/49}$

$x=\pm\frac{2\sqrt{6}}{7}$ quad IV $\therefore x=\frac{2\sqrt{6}}{7}$

e) $x^2+1/9=1$ $x^2=8/9$ $x=\pm\frac{2\sqrt{2}}{3}$ $x<0$ $\therefore x=-\frac{2\sqrt{2}}{3}$

f) $\frac{144}{169}+y^2=1$ $y^2=25/169$ $y=\pm 5/13$ x is (+) $\therefore$ quad IV

$\therefore y=-5/13$

4. a) $(-1,0)$ b) $(0,-1)$ c) $(\frac{1}{2},\frac{\sqrt{3}}{2})$ d) $(\frac{\sqrt{3}}{2},-\frac{1}{2})$

e) $(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2})$ f) $(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2})$ g) $(1,0)$ h) $(0,1)$

i) $(-\sqrt{3}/2,\frac{1}{2})$ j) $(-\frac{1}{2},\frac{\sqrt{3}}{2})$

5. a) $270°, 3\pi/2$ b) $0°, 360°$ $0, 2\pi$ $0 \le \theta < 2\pi$ $\therefore 0°$ or 0

c) $45°, \pi/4$ d) $135°, 3\pi/4$ e) $60°, \pi/3$ f) $300°, 5\pi/3$

g) $150°, 5\pi/6$ h) $210°, 7\pi/6$ i) $225°, 5\pi/4$ j) $180°, \pi$

6. $150°, -210°$ $5\pi/6, -7\pi/6$

7 a) $30°\ (\sqrt{3}/2, 1/2)$ $60°\ (\frac{1}{2}, \frac{\sqrt{3}}{2})$ $45°\ (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
$210°\ (-\sqrt{3}/2, -1/2)$ $240°\ (-\frac{1}{2}, -\frac{\sqrt{3}}{2})$ $225°\ (-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$

b) $120°\ (-1/2, \sqrt{3}/2)$
$300°\ (1/2, -\sqrt{3}/2)$

rotation of 180° changes both sides. (half rotations)

8. quarter rotations (90°)

$30°\ (\sqrt{3}/2, 1/2)$ $(x,y) \to (-y, x)$
$120°\ (-1/2, \sqrt{3}/2)$ " "
$210°\ (-\sqrt{3}/2, -1/2)$
$300°\ (1/2, -\sqrt{3}/2)$

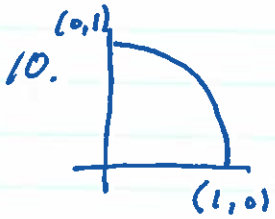
90° rotation
$(x,y) \to (-y, x)$
$-90°: (x,y) \to (y, -x)$

9 a) center $(0,0)$ $r = 1$ $x^2 + y^2 = 1$

b) $(\frac{\sqrt{5}}{3}, \frac{2}{3})$ c) $\hat{AC} = \theta + 90°$ or $\theta + \pi/2$

d) quad IV e) $-1 \le x \le 1$ $-1 \le y \le 1$

10.



a) $0 \le x \le 1$ b) $y^2 = 1 - (.807)^2$
$y^2 = .348751$
$y \approx .591$
$(.807, .591)$

c) $x^2 = 1 - .2571^2$
$x^2 = .9339$
$x \approx .9664$ $(.9664, .2571)$

12 a)  3 rotations -2π to 4π
$-360°$ to $720°$

b) $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$ $120°$: $-240°, 120°, 480°$
radians: $-\frac{4\pi}{3}, \frac{2\pi}{3}, \frac{8\pi}{3}$

c) same terminal point

13 a) $\frac{1}{9} + \frac{4\sqrt{9}}{9} = 1$ ∴ a point on the unit circle
b) $(-, -)$ ∴ quad III
c) $(-Y, x)$ $(\frac{2\sqrt{2}}{3}, -\frac{1}{3})$ d) $(-\frac{2\sqrt{2}}{3}, \frac{1}{3})$

15 a) A (a, b) B $(-a, b)$ C $(-a, -b)$ D $(a, -b)$

b) (i) C (ii) C (iii) B (iv) B

c) same: arc + angle are the same for unit circle

16. a) $225°$, $5\pi/4$
b) $\frac{13\pi}{2}$: A $\pi/2 = 90°$ start at $0°$ & count $13 \times 90°$ turns.
c) $5 \cdot \frac{180}{\pi} \simeq 286°$ ∴ between C and D

17 a) $Y = -3X$ $x^2 + y^2 = 1$
$x^2 + (-3x)^2 = 1$
$10x^2 = 1$
$x^2 = 1/10$
$x = \pm\sqrt{\frac{1}{10}} = \pm\frac{1}{\sqrt{10}} = \pm\frac{\sqrt{10}}{10}$ * $Y = -3X$
$(\frac{\sqrt{10}}{10}, -\frac{3\sqrt{10}}{10})$
$(-\frac{\sqrt{10}}{10}, \frac{3\sqrt{10}}{10})$

b) let $(\frac{\sqrt{10}}{10}, -\frac{3\sqrt{10}}{10}) = P(\theta + \pi)$
then $P(\theta) = (\frac{-\sqrt{10}}{10}, \frac{3\sqrt{10}}{10})$