11.1 Permutations

permutations: order matters
combinations: order doesn't matter

ex 1 a) order matters (jobs are different)

Job 1	A	B	C	D	
Job 2	B C D	A C D	A B D	A B C	12 ways

4 choices for Job 1 × 3 choices for Job 2 = 12 ways.

b) B _ _ _ _ _ B

3 choices 2 choices

$3 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 = 720$ ways.

* $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5!$ (factorial) (MATH → NUM → PRB → !)
$0! = 1$ (defined)

* Permutation ${}_nP_r = \frac{n!}{(n-r)!}$ r items from a set of n items
** if order matters

10 runners in a race → how many possible gold-silver-bronze finishers are there?

$10 \cdot 9 \cdot 8 = 720$

${}_{10}P_3 = \frac{10!}{(10-3)!} = \frac{10!}{7!} = 720$

MATH → PRB → nPr ${}_{10}P_3 = 720$

ex 2 pg 519

a) ${}_9P_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 9 \cdot 8 \cdot 7 \cdot 6 = 3024$

b) $100! + 99! = 100 \cdot 99! + 1 \cdot 99! = 101(99!)$

c) ${}_nP_3 = 60 \quad 60 = \frac{n!}{(n-3)!} \quad 60 = \frac{n(n-1)(n-2)(n-3)!}{(n-3)!}$

$60 = n(n-1)(n-2)$

$n^3 - 3n^2 + 2n = 60$

$n^3 - 3n^2 + 2n - 60 = 0$ check 1, 2, 3, 4, 5, 6, 10, 12...

$5^3 - 3(5)^2 + 2(5) - 60 = 0$ ✓

$n = 5$

* try $\frac{100!}{98!}$ using calculator $\frac{100 \cdot 99 \cdot 98!}{98!} = 100 \cdot 99 = 9900$

page 521 - repeats you can arrange ABCD 4! or 24 ways

but if it was ABBC with B repeated twice

$\frac{4!}{2!} = 12$ ways

MISSISSIPPI → $\frac{11!}{4!4!2!} = 34650$ ways.

ex3 pg 521 a) aardvark $\frac{8!}{3!2!} = 3360$

b) 4 up, 6 right (10 moves) $\frac{10!}{4!6!} = 210$

ex4 pg 522

a) $\frac{4}{} \cdot \frac{3}{} \cdot 1 \cdot \frac{2}{} \cdot \frac{1}{} = 24$

b)

A	B	3	2	1
3	A	B	2	1
3	2	A	B	1
3	2	1	A	B

24 and reverse BA = 24 ∴ 48

c) $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 =$ total − 48 (from b)

$120 - 48$ ∴ 72

ex 5

3 digit number bigger than 300, even, using 1, 2, 3, 4, 5, 6

case 1 → start with odd ≥ 3 (3, 5) 2
(ends with even) (2, 4, 6) 3
(4 others) 4 $2 \cdot 4 \cdot 3 = 24$

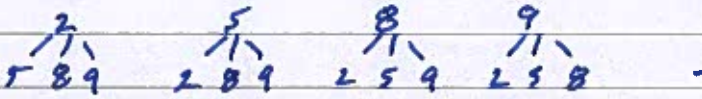
case 2 → start with even ≥ 3 (4, 6) 2
(ends with even) 2 $2 \cdot 4 \cdot 2 = 16$
(4 others) 4

total. 40

exercises pgs 524-527

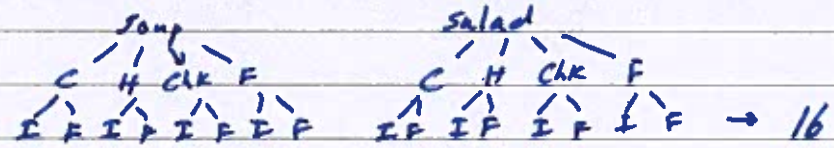
1 a) JAM JMA MAJ MJA AJM AMJ = 6

b)



→ 12

c)



→ 16

2 a) ${}_8P_2 = 56$ b) ${}_7P_5 = 2520$ c) ${}_6P_6 = 720$ d) ${}_4P_1 = 4$

MATH → PAB → nPr

* 2nd entry (typeover)

3. $4! + 3! = 4 \cdot 3 \cdot 2 \cdot 1 + 3 \cdot 2 \cdot 1 = 24 + 6 = 30$

$(4+3)! = 7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$

4 a) $9! = 362880$ b) $\frac{362880}{120 \cdot 24} = 126$ c) $120 \cdot 6 = 720$ d) $6 \cdot 24 = 144$

e) $\frac{102 \cdot 101 \cdot 100!}{100!\, 2!} = \frac{102 \cdot 101}{2} = 5151$ f) $7! - 5! = 4920$

5 a) $\frac{6!}{2!} = 360$ b) $\frac{7!}{3!\, 2!} = 420$ c) $\frac{11!}{4!\, 3!\, 2!} = 138600$

5d) $\frac{6!}{3!3!} = 20$ e) $\frac{5!}{3!} = 20$ f) $\frac{8!}{2!2!} = 10080$

6. $4! = 24$

7a) $\frac{n!}{(n-2)!} = 30$ $\frac{n(n-1)(n-2)!}{(n-2)!} = 30$ $n(n-1) = 30$

$n^2 - n - 30 = 0$

$(n-6)(n+5) = 0$

$n = 6$ ~~$n = -5$~~ $n = 6$

b) $\frac{n(n-1)(n-2)(n-3)!}{(n-3)!} = 990$ $n^3 - 3n^2 + 2n = 990$

$n^3 - 3n^2 + 2n - 990 = 0$ $n = 11$

(just checked table) →

c) $\frac{6!}{(6-r)!} = 30$ $\frac{6!}{30} = (6-r)!$ $24 = (6-r)!$

$6 - r = 4$

$r = 2$

d) ${}_nP_2 = 30$ $\frac{n!}{(n-2)!} = 30$ $n(n-1) = 30$ (same as (a)) $n = 6$

8. a) 4 moves 2 down 2 right $\frac{4!}{2!2!} = 6$

b) 7 moves 3 up 4 right $\frac{7!}{4!3!} = 35$

c) 5 moves 2 up 3 left $\frac{5!}{2!3!} = 10$

9. a) start with odd (3,5) 2 start with even (2,4) 2

end even (2,4) 2 end even 1

3 left $\frac{3}{12}$ 3 left $\frac{3}{6}$ ∴ 18

b) start with B 1 start with E 1

end with vowel 3 end with vowel 2

4 other $\frac{4}{12}$ 4 other $\frac{4}{8}$ ∴ 20

10 a) $2 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 = 48$

b) $B_1\ B_2 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ — 24
$4\ B_1\ B_2\ 3\ 2\ 1$ — 24
$4\ 3\ B_1\ B_2\ 2\ 1$ — 24
$4\ 3\ 2\ B_1\ B_2\ 1$ — 24
$4\ 3\ 2\ 1\ B_1\ B_2$ — 24

reverse B_1, B_2

$120 \times 2 = 240$

* treat $B_1 B_2$ as one value $5! \cdot 2 = 240$

c) $4\ 3\ B_1\ B_2\ 2\ 1 = 24$
$4\ 3\ B_2\ B_1\ 2\ 1 = 24$ ∴ 48

11 a) $7! = 5040$ b) $\frac{7!}{2!} = 2520$ c) $M \cdot 6!$ 720 ∴ 1440
$6!\ M$ 720

d) $4! = 24$
↑ count as 1 + 3 others $4! \cdot 24 = 576$

12. $6! = 720$ $4 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 = ~~384~~ 288$

13. ~~$9 \cdot 9 \cdot 8$~~ ← numbers ~~$25 \cdot 25 \cdot 24 \cdot 23$~~ ← letters.
$25 \cdot 10 \cdot 9 \cdot 8 = 18000$ (not enough)
↑ letter ↑↑↑ digits

14 $5 \cdot 4 = 20$

15. $40 \cdot 40 \cdot 40 = 64000$ possibilities $\div$ 4/min = 16000 min $\div$ 60 min/hr
$= 266\frac{2}{3}$ hr

16. a) $7! = 5040$ b) $2! \cdot 6! = 1440$ c) together = 1440 (like b)
$5040 - 1440$ (total − together)
3600

17 a) $\frac{8!}{3!2!} = 3360$ b) (L L L) $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ / 2

$\frac{6!}{2} = 360$

18 5 letter, 2 sets of doubles AABBC

b) level

19. $\frac{5}{34\,56} \cdot \frac{9}{10\,7} \cdot \frac{9}{10\,7} \cdot \frac{9}{10\,7} = 3645$

20 a) $26 \cdot 10 \cdot 26 \cdot 10 \cdot 26 \cdot 10 = 17\,576\,000$ postal codes

b) I don't think Canada will have more than 17576000 communities

21 a) 10 pgs · 1 sonnet · 14 lines 10 choices for line 1

10 choices for line 2 ... 10^{14}

b) 100 million million

$10^2 \cdot 10^6 \cdot 10^6 = 10^{14}$ ✓

22 a) ${}_3P_r = 3!$ $\frac{3!}{(3-r)!} = 3!$ $(3-r)! = 1$ $r = 2$ or 3

$1! = 1$

$0! = 1$

b) ${}_7P_r = 7!$ $\frac{7!}{(7-r)!} = 7!$ $(7-r)! = 1$ $r = 6$ or 7

c) ${}_nP_3 = 4({}_{n-1}P_2)$ $\frac{n!}{(n-3)!} = \frac{4(n-1)!}{(n-1-2)!}$ $n! = 4(n-1)!$

$n(n-1)! = 4(n-1)!$

$n = 4$

d) ${}_n5P_3 = {}_7P_5$ $n = \frac{{}_7P_5}{5P_3} = \frac{2520}{60}$ $n = 42$

23 ${}_nP_n = n!$ ${}_nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!}$ $\therefore 0! = 1$

24. ${}_3P_5$ ← arranging 5 items
↑ 3 items

(like arranging top 5 finishers in a race with only 3 runners)

25. start with 1,3,5 → 3 · finish odd → 2 · 4 middle → 4 · 24

start with 2,4 → 2 · finish odd → 3 · 4 middle → 4 · 24 ∴ 48

} 3 digits

2 digits: start with 1,3,5 → 3 · finish odd → 2 · 6

start with 2,4 → 2 · finish odd → 3 · 6 ∴ 12

} 2 digits

1 digit: 1,3,5 = 3

total = 63

26. ~~start 2 · 3! · 2 = 6~~ ~~5 digit · 5 digit (can't 0,2) · 3 · 2 · 1 · 2 = 48~~ ~~not 0~~

~~4 digit start 2~~

~~1 · 3 · 2 · 1 = 6~~

5 digit (starting with 2) $1 \cdot 3 \cdot 2 \cdot 1 \cdot 1 = 6$

↑ must be 2 ↑ must be 0

5 digit (starting 1,3,5) $3 \cdot 3 \cdot 2 \cdot 1 \cdot 2 = 36$

4 digit (starting 2) $1 \cdot 3 \cdot 2 \cdot 1 = 6$

4 digit (starting 1,3,5) $3 \cdot 3 \cdot 2 \cdot 2 = 36$ total 84

27. 1 - 9 → 9 ("between" so 1 doesn't count) → 8

10 - 99 → 90 numbers − 9 repeats → 81

3 digits $9 \cdot 9 \cdot 8 = 648$ → 648

↑ 1-9 ↑ 0-9 ↑ 0-9

no repeat no repeat

737

28. $\frac{6!}{4! \, 2!} = 15$ 6 spots

4 blue 2 yellow