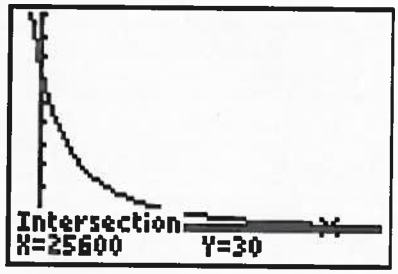
c) Graph $W_h = \dfrac{750}{\left(\dfrac{h}{6400}+1\right)^2}$ and $W_h = 30$ and find the point of intersection.



So, $h > 25\,600$ or a height of more than 25 600 km will result in the astronaut having a weight of less than 30 N.

Cumulative Review, Chapters 5–7

Cumulative Review **Page 416** **Question 1**

$$\begin{aligned}3xy^3\sqrt{2x} &= \sqrt{(3xy^3)^2(2x)}\\ &= \sqrt{3^2x^2y^6(2x)}\\ &= \sqrt{18x^3y^6}\end{aligned}$$

Cumulative Review **Page 416** **Question 2**

$$\begin{aligned}\sqrt{48a^3b^2c^5} &= \sqrt{16(3)(a^2)(a)(b^2)(c^4)(c)}\\ &= 4abc^2\sqrt{3ac}\end{aligned}$$

Cumulative Review **Page 416** **Question 3**

$3\sqrt{6} = \sqrt{9(6)}$ $\quad\sqrt{36}\quad$ $2\sqrt{3} = \sqrt{4(3)}$ $\quad\sqrt{18}\quad$ $2\sqrt{9} = \sqrt{4(9)}$ $\quad\sqrt[3]{8} = 2$

$= \sqrt{54}$ $\quad= \sqrt{12}$ $\quad= \sqrt{36}$ $\quad= \sqrt{4}$

The numbers from least to greatest are $\sqrt[3]{8}, 2\sqrt{3}, \sqrt{18}, \sqrt{36}, 2\sqrt{9},$ and $3\sqrt{6}$ or $\sqrt[3]{8}, 2\sqrt{3}, \sqrt{18}, 2\sqrt{9}, \sqrt{36},$ and $3\sqrt{6}$.

Cumulative Review **Page 416** **Question 4**

a) $4\sqrt{2a} + 5\sqrt{2a} = 9\sqrt{2a},\ a \geq 0$

b) $$\begin{aligned}10\sqrt{20x^2} - 3x\sqrt{45} &= 20x\sqrt{5} - 9x\sqrt{5}\\ &= 11x\sqrt{5}\end{aligned}$$

Cumulative Review **Page 416** **Question 5**

a) $2\sqrt[3]{4}\left(-4\sqrt[3]{6}\right) = -8\sqrt[3]{24}$

$= -16\sqrt[3]{3}$

b) $\sqrt{6}\left(\sqrt{12} - \sqrt{3}\right) = \sqrt{72} - \sqrt{18}$

$= 6\sqrt{2} - 3\sqrt{2}$

$= 3\sqrt{2}$

c) $\left(6\sqrt{a} + \sqrt{3}\right)\left(2\sqrt{a} - \sqrt{4}\right) = 12a - 6\sqrt{4a} + 2\sqrt{3a} - \sqrt{12}$

$= 12a - 12\sqrt{a} + 2\sqrt{3a} - 2\sqrt{3}, a \geq 0$

Cumulative Review **Page 416** **Question 6**

a) $\dfrac{\sqrt{12}}{\sqrt{4}} = \dfrac{\sqrt{12}}{\sqrt{4}}\left(\dfrac{\sqrt{4}}{\sqrt{4}}\right)$

$= \dfrac{\sqrt{48}}{4}$

$= \dfrac{4\sqrt{3}}{4}$

$= \sqrt{3}$

b) $\dfrac{2}{2+\sqrt{3}} = \dfrac{2}{2+\sqrt{3}}\left(\dfrac{2-\sqrt{3}}{2-\sqrt{3}}\right)$

$= \dfrac{4-2\sqrt{3}}{4-3}$

$= 4 - 2\sqrt{3}$

c)

$\dfrac{\sqrt{7}+\sqrt{28}}{\sqrt{7}-\sqrt{14}} = \dfrac{\sqrt{7}+\sqrt{28}}{\sqrt{7}-\sqrt{14}}\left(\dfrac{\sqrt{7}+\sqrt{14}}{\sqrt{7}+\sqrt{14}}\right)$

$= \dfrac{7+\sqrt{7(14)}+\sqrt{7(28)}+\sqrt{28(14)}}{7-14}$

$= \dfrac{7+7\sqrt{2}+14+14\sqrt{2}}{-7}$

$= \dfrac{21+21\sqrt{2}}{-7}$

$= -3 - 3\sqrt{2}$

Cumulative Review Page 416 Question 7

$$\sqrt{x+6} = x,\ x \ge -6$$
$$\left(\sqrt{x+6}\right)^2 = x^2$$
$$x+6 = x^2$$
$$0 = x^2 - x - 6$$
$$0 = (x-3)(x+2)$$

$x - 3 = 0$ or $x + 2 = 0$

$x = 3$ $\quad x = -2$

Check for $x = 3$.

Left Side	Right Side
$\sqrt{x+6}$	x
$= \sqrt{3+6}$	$= 3$
$= 3$	

Left Side = Right Side

The solution is $x = 3$.

Check for $x = -2$.

Left Side	Right Side
$\sqrt{x+6}$	x
$= \sqrt{-2+6}$	$= -2$
$= 2$	

Left Side ≠ Right Side

Cumulative Review Page 416 Question 8

a) Substitute $v = 20$ and $r = 15$ into $v = \sqrt{h-2r}$ and solve for h.

$$v = \sqrt{h-2r}$$
$$20 = \sqrt{h-2(15)}$$
$$20^2 = \left(\sqrt{h-30}\right)^2$$
$$400 = h - 30$$
$$h = 430$$

The height of the fill is 430 ft.

b) Example: The velocity would decrease with an increasing radius because the expression $\sqrt{h-2r}$ would decrease.

Cumulative Review Page 416 Question 9

a) $\dfrac{12a^3b}{48a^2b^4} = \dfrac{a}{4b^3},\ a \ne 0, b \ne 0$

b) $\dfrac{4-x}{x^2-8x+16} = \dfrac{-(x-4)}{(x-4)(x-4)}$

$= -\dfrac{1}{x-4},\ x \ne 4$

c) $\dfrac{(x-3)(x+5)}{x^2-1} \div \dfrac{x+2}{x-3} = \dfrac{(x-3)(x+5)}{(x-1)(x+1)}\left(\dfrac{x-3}{x+2}\right)$

$$= \frac{(x-3)^2(x+5)}{(x-1)(x+1)(x+2)}, \quad x \neq -2, -1, 1, 3$$

d) $\dfrac{5x-10}{6x} \times \dfrac{3x}{15x-30} = \dfrac{\cancel{5}\,\cancel{(x-2)}}{\cancel{6x}_{\,2}}\left(\dfrac{\cancel{3x}}{\cancel{15}_{\,3}\,\cancel{(x-2)}}\right)$

$$= \frac{1}{6}, \quad x \neq 0, 2$$

e) $\left(\dfrac{x+2}{x-3}\right)\left(\dfrac{x^2-9}{x^2-4}\right) \div \left(\dfrac{x+3}{x-2}\right) = \left(\dfrac{\cancel{x+2}}{\cancel{x-3}}\right)\left(\dfrac{\cancel{(x-3)}\,\cancel{(x+3)}}{\cancel{(x-2)}\,\cancel{(x+2)}}\right)\left(\dfrac{\cancel{x-2}}{\cancel{x+3}}\right)$

$$= 1, \quad x \neq -3, -2, 2, 3$$

Cumulative Review Page 416 Question 10

a) $\dfrac{10}{a+2} + \dfrac{a-1}{a-7} = \dfrac{10(a-7)}{(a+2)(a-7)} + \dfrac{(a-1)(a+2)}{(a-7)(a+2)}$

$$= \frac{10a - 70 + a^2 + a - 2}{(a+2)(a-7)}$$

$$= \frac{a^2 + 11a - 72}{(a+2)(a-7)}$$

$$= \frac{a^2 + 11a - 72}{(a+2)(a-7)}, \quad a \neq -2, 7$$

b) $\dfrac{3x+2}{x+4} - \dfrac{x-5}{x^2-4} = \dfrac{(3x+2)(x-2)(x+2)}{(x+4)(x-2)(x+2)} - \dfrac{(x-5)(x+4)}{(x-2)(x+2)(x+4)}$

$$= \frac{3x^3 - 12x + 2x^2 - 8 - x^2 + x + 20}{(x+4)(x-2)(x+2)}$$

$$= \frac{3x^3 + x^2 - 11x + 12}{(x+4)(x-2)(x+2)}, \quad x \neq -4, -2, 2$$

c) $\dfrac{2x}{x^2-25}-\dfrac{3}{x^2-4x-5}=\dfrac{2x(x+1)}{(x+5)(x-5)(x+1)}-\dfrac{3(x+5)}{(x-5)(x+1)(x+5)}$

$$=\frac{2x^2+2x-3x-15}{(x+5)(x-5)(x+1)}$$

$$=\frac{2x^2-x-15}{(x+5)(x-5)(x+1)},\quad x\neq -5,-1,5 \quad \text{or}$$

$$=\frac{(2x+5)(x-3)}{(x+5)(x-5)(x+1)},\quad x\neq -5,-1,5$$

Cumulative Review Page 416 Question 11

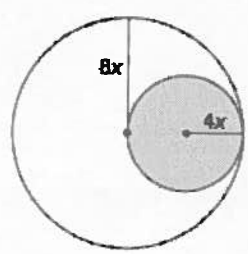
Example: I disagree with Sandra. The expression $\dfrac{(x+2)(x+5)}{x+5}$ is not equivalent to $x + 2$. She forgot to state the restriction on the variable. The expression $\dfrac{(x+2)(x+5)}{x+5}$ is equivalent to $x + 2$, $x \neq -5$.

Cumulative Review Page 416 Question 12

$$\frac{x+4}{4}=\frac{x}{3}$$
$$12\left(\frac{x+4}{4}\right)=\left(\frac{x}{3}\right)12$$
$$3(x+4)=4x$$
$$12=x$$
The value of x is 12.

Cumulative Review Page 417 Question 13

$$P=\frac{\text{area of shaded region}}{\text{area of entire figure}}$$



$$P=\frac{\pi(4x)^2}{\pi(8x)^2}$$
$$P=\frac{16\pi x^2}{64\pi x^2}, x\neq 0$$
$$P=\frac{1}{4}$$

The probability that the point is in the shaded region is $\frac{1}{4}$.

Cumulative Review   Page 417   Question 14

First, evaluate each number and express it in decimal form.

$|-5| = 5$

$|4 - 6| = |-2|$
$= 2$

$|2(-4) - 5| = |-8 - 5|$
$= |-13|$
$= 13$

$|8.4| = 8.4$

The numbers from least to greatest are $|4 - 6|$, $|-5|$, $|8.4|$, and $|2(-4) - 5|$.

Cumulative Review   Page 417   Question 15

a) The x-intercept is $x = 2$. When $x \geq 2$, the graph of $y = |3x - 6|$ is the graph of $y = 3x - 6$. When $x < 2$, the graph of $y = |3x - 6|$ is the graph of $y = -(3x - 6)$ or $y = -3x + 6$. The absolute value function $y = |3x - 6|$ expressed as a piecewise function is

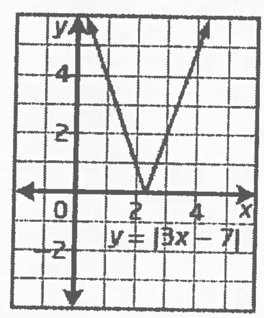
$$y = \begin{cases} 3x - 6, \text{ if } x \geq 2 \\ -3x + 6, \text{ if } x < 2 \end{cases}.$$

b) The x-intercepts are $x = -1$ and $x = 5$. When $x \leq -1$ or $x \geq 5$, the graph of $y = \left|\frac{1}{3}(x-2)^2 - 3\right|$ is the graph of $y = \frac{1}{3}(x-2)^2 - 3$. When $-1 < x < 5$, the graph of $y = \left|\frac{1}{3}(x-2)^2 - 3\right|$ is the graph of $y = -\left(\frac{1}{3}(x-2)^2 - 3\right)$ or $y = -\frac{1}{3}(x-2)^2 + 3$. The absolute value function $y = \left|\frac{1}{3}(x-2)^2 - 3\right|$ expressed as a piecewise function is

$$y = \begin{cases} \frac{1}{3}(x-2)^2 - 3, \text{ if } x \leq -1 \text{ or } x \geq 5 \\ -\frac{1}{3}(x-2)^2 + 3, \text{ if } -1 < x < 5 \end{cases}.$$

Cumulative Review   Page 417   Question 16

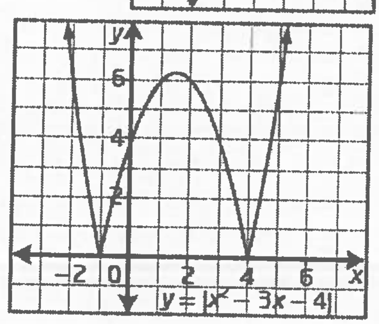
a) i) Use a table of values to sketch the graph.



ii) x-intercept: $\frac{7}{3}$, y-intercept: 7

iii) domain: $\{x \mid x \in R\}$, range: $\{y \mid y \geq 0, y \in R\}$

b) i) Use a table of values to sketch the graph.



ii) x-intercepts: -1 and 4, y-intercept: 4

iii) domain: $\{x \mid x \in R\}$, range: $\{y \mid y \geq 0, y \in R\}$

Cumulative Review Page 417 Question 17

a) Examine the two cases.
Case 1

The expression $|2x-1|$ equals $2x-1$ when $x \geq \frac{1}{2}$.

$$\begin{aligned} 2x - 1 &= 9 \\ 2x &= 10 \\ x &= 5 \end{aligned}$$

The value 5 satisfies the condition $x \geq \frac{1}{2}$.

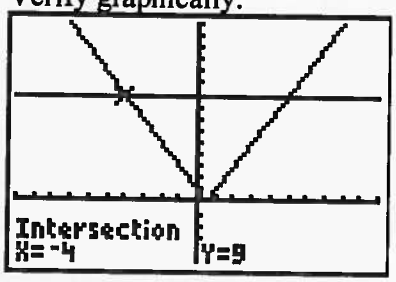
Case 2

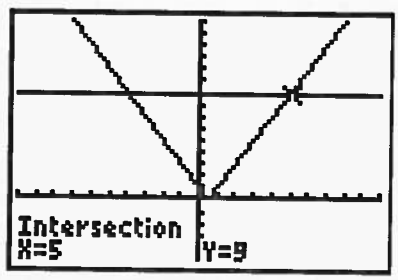
The expression $|2x-1|$ equals $-(2x-1)$ when $x < \frac{1}{2}$.

$$\begin{aligned} -(2x - 1) &= 9 \\ 2x - 1 &= -9 \\ 2x &= -8 \\ x &= -4 \end{aligned}$$

The value -4 satisfies the condition $x < \frac{1}{2}$.

The solution is $x = -4$ or $x = 5$.
Verify graphically.





b) Examine the two cases.
Case 1

The expression $|2x^2-5|$ equals $2x^2-5$ when $x \leq -\sqrt{\frac{5}{2}}$ or $x \geq \sqrt{\frac{5}{2}}$ or approximately $x \leq -1.58$ or $x \geq 1.58$.

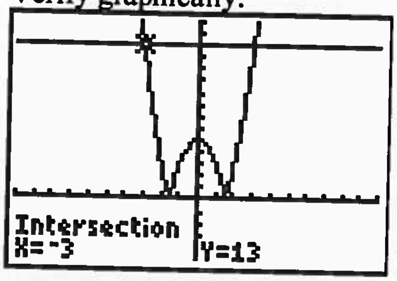
$$\begin{aligned} 2x^2 - 5 &= 13 \\ 2x^2 &= 18 \\ x^2 &= 9 \\ x &= \pm 3 \end{aligned}$$

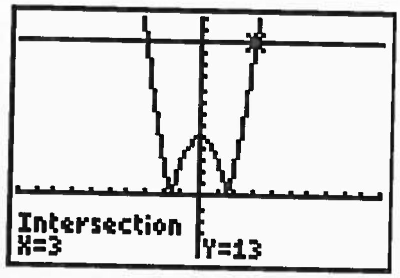
Both values satisfy the conditions.
Case 2

The expression $|2x^2-5|$ equals $-(2x^2-5)$ when $-\sqrt{\frac{5}{2}} < x < \sqrt{\frac{5}{2}}$.

$-(2x^2 - 5) = 13$
$2x^2 - 5 = -13$
$2x^2 = -8$
$x^2 = -4$
There are no solutions.

The solution is $x = -3$ or $x = 3$.
Verify graphically.





Cumulative Review **Page 417** **Question 18**

a) Example: Absolute value must be used in the formula for area because area cannot be negative.

b) Substitute $a = -5$, $b = 2$, $c = -3$, and $d = 4$ into $A = \frac{1}{2}|ad - bc|$.

$A = \frac{1}{2}|ad - bc|$

$A = \frac{1}{2}|-5(4) - 2(-3)|$

$A = \frac{1}{2}|-20 + 6|$

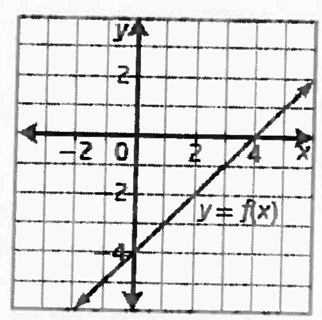
$A = \frac{1}{2}|-14|$

$A = \frac{1}{2}(14)$

$A = 7$

The area of the triangle is 7 square units.

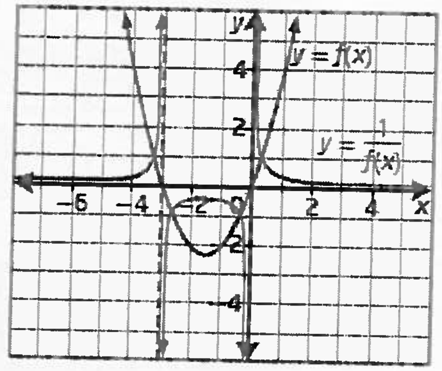
Cumulative Review **Page 417** **Question 19**



Since the graph of the reciprocal function has a vertical asymptote of $x = 4$, the graph of the original function will have x-intercept at (4, 0). The given point, (5, 1), is an invariant point so it is also a point on the graph of the original function. Plot the two points and draw the line.
$f(x) = x - 4$

Cumulative Review Page 417 Question 20

Example: Locate zeros of the original function (where $y = 0$) and draw the vertical asymptotes. Locate the invariant points (where $f(x)$ has a value of 1 or –1). Use these points to help sketch the graph of the reciprocal function.



Cumulative Review Page 417 Question 21

Determine characteristics of the graph of $f(x)$.

- For $f(x) = (x + 2)^2$, the coordinates of the vertex are (–2, 0).
- Determine the zeros of the function, or x-intercepts of the graph, by solving $f(x) = 0$.

$(x + 2)^2 = 0$
$x + 2 = 0$
$x = -2$

- The y-intercept is 0.

To sketch the graph of the reciprocal function, consider the following characteristics:

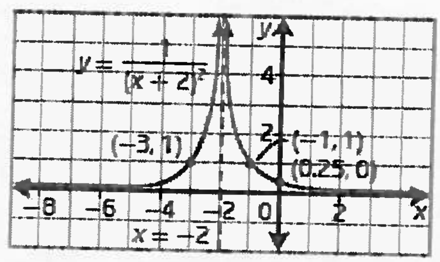
- The reciprocal function has vertical asymptote $x = -2$.
- Determine the invariant points by solving $f(x) = \pm 1$.

$(x + 2)^2 = 1$
$x + 2 = \pm 1$
$x + 2 = -1$ or $x + 2 = 1$
$x = -3$ $x = -1$

$(x + 2)^2 = -1$
No solutions.

Invariant points are (–3, 1) and (–1, 1).

The y-intercept of $y = \dfrac{1}{(x+2)^2}$ is $\dfrac{1}{4}$ or 0.25.



Cumulative Review Page 417 Question 22

a) Example: The shape, range, and y-intercept will be different for $y = |f(x)|$ from $f(x) = 3x - 1$.

For $f(x) = 3x - 1$, the graph is a line, the range is $\{y \mid y \in \mathrm{R}\}$, and the y-intercept is –1.
For $y = |3x - 1|$, the graph is vee-shaped, the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$, and the y-intercept is 1.

b) Example: The graph of the reciprocal function has a horizontal asymptote, a vertical asymptote, and no x-intercept.

For $f(x) = 3x - 1$, the graph is a line, the x-intercept is $\frac{1}{3}$, and the y-intercept is -1.

For $y = \frac{1}{3x-1}$, the graph has two branches, a horizontal asymptote at $y = 0$, and a vertical asymptote at $x = \frac{1}{3}$, and the y-intercept is -1.

Unit 3 Test

Unit 3 Test Page 418 Question 1

$$\begin{aligned} 2\sqrt[3]{-27} &= \sqrt[3]{2^3(-27)} \\ &= \sqrt[3]{8(-27)} \\ &= \sqrt[3]{-216} \end{aligned}$$

The best answer is **C**.

Unit 3 Test Page 418 Question 2

$$\begin{aligned} \frac{4\sqrt{72x^5}}{x\sqrt{8}} &= \frac{4}{x}\left(\sqrt{\frac{72x^4(x)}{8}}\right) \\ &= \frac{4x^2}{x}\left(\sqrt{9x}\right) \\ &= 12x\left(\sqrt{x}\right) \end{aligned}$$

The best answer is **D**.

Unit 3 Test Page 418 Question 3

$$\begin{aligned} x+2 &= \sqrt{x^2+3} \\ (x+2)^2 &= x^2+3 \\ x^2+4x+4 &= x^2+3 \\ 4x &= -1 \\ x &= -\frac{1}{4} \end{aligned}$$

The best answer is **B**.