$2x - 3 = 0$ or $x + 9 = 0$

$x = \frac{3}{2}$ $\quad$ $x = -9$

Since width must be positive, the grass strip will be 1.5 m in width.
Example: Factoring is a quick method.

c) The outside dimensions are 12 m by 9 m. So, the perimeter of the outside of the path is 42 m.

Cumulative Review, Chapters 3–4

Cumulative Review Page 264 Question 1

a) For a vertex in quadrant III, both the x-coordinate and the y-coordinate must be negative. The quadratic function $y = 2(x + 2)^2 - 3$ meets this requirement. Choose **C**.

b) For a parabola that opens downward, $a < 0$.. The quadratic function $y = -5(x - 2)^2 - 3$ meets this requirement. Choose **A**.

c) For an axis of symmetry of $x = 3$, the value of p is 3. The quadratic function $y = 3(x - 3)^2 - 5$ meets this requirement. Choose **D**.

d) For a range of $\{y \mid y \geq 5, y \in \mathrm{R}\}$, $a > 0$ and $q = 5$. The quadratic function $y = 3(x + 3)^2 + 5$ meets this requirement. Choose **B**.

Cumulative Review Page 264 Question 2

a) The function $y = (x - 6) - 1$ has degree 1. It is not a quadratic function.

b) The function $y = -5(x + 1)^2$ has degree 2. It is a quadratic function.

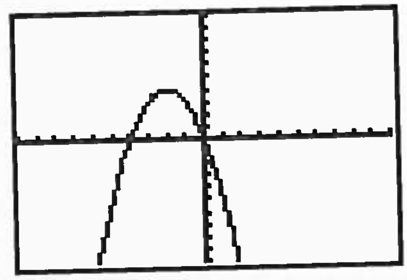
c) The function $y = \sqrt{(x+2)^2} + 7$ has degree 1. It is not a quadratic function.

d) The function $y + 8 = x^2$ has degree 2. It is a quadratic function.

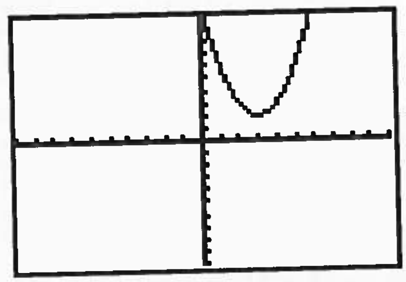
Cumulative Review Page 264 Question 3

Examples:

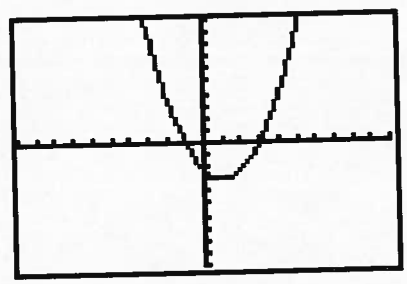
a) For an axis of symmetry with equation $x = -2$ and a range of $\{y \mid y \le 4, y \in R\}$, the graph must have a vertex at (–2, 4) and open downward.



b) For an axis of symmetry with equation $x = 3$ and a range of $\{y \mid y \ge 2, y \in R\}$, the graph must have a vertex at (3, 2) and open upward.



c) For a parabola that opens upward with vertex at (1, –3) and an x-intercept at (3, 0), the other x-intercept must be at (–1, 0) and have a range of $\{y \mid y \ge -3, y \in R\}$.



Cumulative Review Page 264 Question 4

a) For $f(x) = (x + 4)^2 - 3$, $a = 1$, $p = -4$, and $q = -3$.
The vertex is located at (–4, –3).
The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \ge -3, y \in R\}$.
The equation of the axis of symmetry is $x = -4$.
The x-intercepts are approximately –5.7 and –2.3.
The y-intercept is $(0 + 4)^2 - 3$, or 13.

b) For $f(x) = -(x - 2)^2 + 1$, $a = -1$, $p = 2$, and $q = 1$.
The vertex is located at (2, 1).
The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \le 1, y \in R\}$.
The equation of the axis of symmetry is $x = 2$.
The x-intercepts are 1 and 3.
The y-intercept is $-(0 - 2)^2 + 1$, or –3.

c) For $f(x) = -2x^2 - 6$, $a = -2$, $p = 0$, and $q = -6$.
The vertex is located at (0, –6).
The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \le -6, y \in \mathrm{R}\}$.
The equation of the axis of symmetry is $x = 0$.
Since the graph opens downward ($a < 0$) and has a maximum value of –6, which is below the x-axis, there are no x-intercepts.
The y-intercept is $-2(0)^2 - 6$, or –6.

d) For $f(x) = \frac{1}{2}(x + 8)^2 + 6$, $a = \frac{1}{2}$, $p = -8$, and $q = 6$.
The vertex is located at (–8, 6).
The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \ge 6, y \in \mathrm{R}\}$.
The equation of the axis of symmetry is $x = -8$.
Since the graph opens upward ($a > 0$) and has a minimum value of 6, which is above the x-axis, there are no x-intercepts.
The y-intercept is $\frac{1}{2}(0 + 8)^2 + 6$, or 38.

Cumulative Review Page 264 Question 5

a) Complete the square to write $y = x^2 - 10x + 18$ in vertex form.
$y = x^2 - 10x + 18$
$y = (x^2 - 10x) + 18$
$y = (x^2 - 10x + 25 - 25) + 18$
$y = (x^2 - 10x + 25) - 25 + 18$
$y = (x - 5)^2 - 7$
The graph of $y = (x - 5)^2 - 7$ will have the same shape as the graph of $y = x^2$, since $a = 1$. Since $p = 5$ and $q = -7$, this represents a horizontal translation of 5 units to the right and a vertical translation of 7 units down relative to the graph of $y = x^2$.

b) Complete the square to write $y = -x^2 + 4x - 7$ in vertex form.
$y = -x^2 + 4x - 7$
$y = -(x^2 - 4x) - 7$
$y = -(x^2 - 4x + 4 - 4) - 7$
$y = -(x^2 - 4x + 4) + 4 - 7$
$y = -(x - 2)^2 - 3$
The graph of $y = -(x - 2)^2 - 3$ will have the same shape as the graph of $y = x^2$ but be reflected in the x-axis, since $a = -1$. Since $p = 2$ and $q = -3$, this represents a horizontal translation of 2 units to the right and a vertical translation of 3 units down relative to the graph of $y = x^2$.

c) Complete the square to write $y = 3x^2 - 6x + 5$ in vertex form.
$y = 3x^2 - 6x + 5$
$y = 3(x^2 - 2x) + 5$
$y = 3(x^2 - 2x + 1 - 1) + 5$
$y = 3(x^2 - 2x + 1) - 3 + 5$
$y = 3(x - 1)^2 + 2$
The graph of $y = 3(x - 1)^2 + 2$ will be narrower than the graph of $y = x^2$, since $a > 1$. Since $p = 1$ and $q = 2$, this represents a horizontal translation of 1 unit to the right and a vertical translation of 2 units up relative to the graph of $y = x^2$.

d) Complete the square to write $y = \frac{1}{4}x^2 + 4x + 20$ in vertex form.

$y = \frac{1}{4}x^2 + 4x + 20$

$y = \frac{1}{4}(x^2 + 16x) + 20$

$y = \frac{1}{4}(x^2 + 16x + 64 - 64) + 20$

$y = \frac{1}{4}(x^2 + 16x + 64) - 16 + 20$

$y = \frac{1}{4}(x + 8)^2 + 4$

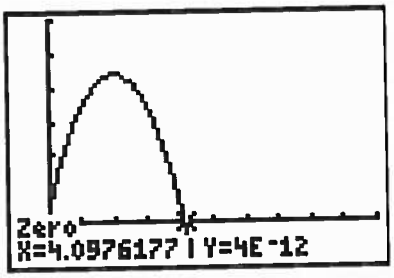
The graph of $y = \frac{1}{4}(x + 8)^2 + 4$ will be wider than the graph of $y = x^2$, since $0 < a < 1$.
Since $p = -8$ and $q = 4$, this represents a horizontal translation of 8 units to the left and a vertical translation of 4 units up relative to the graph of $y = x^2$.

Cumulative Review Page 264 Question 6

a) Complete the square to find the maximum.
$h(t) = -5t^2 + 20t + 2$
$h(t) = -5(t^2 - 4t) + 2$
$h(t) = -5(t^2 - 4t + 4 - 4) + 2$
$h(t) = -5(t^2 - 4t + 4) + 20 + 2$
$h(t) = -5(t - 2)^2 + 22$
The maximum height reached by the arrow is 22 m.

b) The arrow was shot from a height of 2 m.

c) Use a graphing calculator to graph the function with window settings of x: [–1, 10, 1] and y: [–5, 30, 5]. The arrow hits the ground in 4 s, to the nearest second.



Cumulative Review Page 264 Question 7

When solving quadratic equations, you may consider the relationship among the **roots** of a quadratic equation, the **zeros** of the corresponding quadratic function, and the ***x*-intercepts** of the graph of the quadratic function.

Cumulative Review Page 264 Question 8

a) $9x^2 + 6x - 8 = (3x + 4)(3x - 2)$

b) $16r^2 - 81s^2 = (4r - 9s)(4r + 9s$

c) Let $r = x + 1$.

$2(x + 1)^2 + 11(x + 1) + 14$
$= 2r^2 + 11r + 14$
$= (2r + 7)(r + 2)$
$= (2(x + 1) + 7)(x + 1 + 2)$
$= (2x + 9)(x + 3)$

d) Let $r = xy$.

$x^2y^2 - 5xy - 36$
$= r^2 - 5r - 36$
$= (r - 9)(r + 4)$
$= (xy - 9)(xy + 4)$

e) Use the pattern for factoring a difference of squares.

$9(3a + b)^2 - 4(2a - b)^2$
$= [3(3a + b) - 2(2a - b)][3(3a + b) + 2(2a - b)]$
$= (5a + 5b)(13a + b)$
$= 5(a + b)(13a + b)$

f) $121r^2 - 400 = (11r - 20)(11r + 20)$

Cumulative Review Page 264 Question 9

Let x, $x + 1$, and $x + 2$ represent the three consecutive integers. For a sum of squares of the integers equal to 194,

$194 = x^2 + (x + 1)^2 + (x + 2)^2$
$194 = x^2 + x^2 + 2x + 1 + x^2 + 4x + 4$
$0 = 3x^2 + 6x - 189$
$0 = 3(x^2 + 2x - 63)$
$0 = 3(x - 7)(x + 9)$

$x - 7 = 0$ or $x + 9 = 0$

$x = 7$ $x = -9$

The three consecutive integers are 7, 8, and 9 or –9, –8, and –7.

Cumulative Review Page 265 Question 10

Let x represent the number of seats in each row. Then, the number of rows is $x + 4$. For a total of 285 seats,

$$285 = x(x + 4)$$
$$0 = x^2 + 4x - 285$$
$$0 = (x + 19)(x - 15)$$

$x + 19 = 0$ or $x - 15 = 0$

$x = -19$ $x = 15$

Since the number seats must be positive, $x = -19$ is an extraneous root.

There are 15 seats in each row and 19 rows.

Cumulative Review Page 265 Question 11

Let x represent the width of the deck. Then, the radius of the deck and hot tub is $x + 1$. For a total area of 63.6 m^2,

$$\pi(x + 1)^2 = 63.6$$
$$(x + 1)^2 = \frac{63.6}{\pi}$$
$$x + 1 = \pm\sqrt{\frac{63.6}{\pi}}$$
$$x = -1 \pm \sqrt{\frac{63.6}{\pi}}$$

$x = -1 + \sqrt{\frac{63.6}{\pi}}$ or $x = -1 - \sqrt{\frac{63.6}{\pi}}$

$x \approx 3.5$ $x \approx -5.5$

Since the width must be positive, $x = -5.5$ is an extraneous root.

The deck is 3.5 m wide, to the nearest tenth of a metre.

Cumulative Review Page 265 Question 12

Example: Dallas forgot to factor out 2 from the coefficient of the x-term in line 1. In line 2, Dallas should have added 2 times the value added in the brackets to the right side. The correct solution is shown.

$$2(x^2 - 6x) = 7$$
$$2(x^2 - 6x + 9) = 7 + 18$$

$$2(x-3)^2 = 25$$
$$x = 3 \pm \sqrt{\frac{25}{2}}$$
$$x = 3 \pm \frac{5\sqrt{2}}{2}$$
$$x = \frac{6 \pm 5\sqrt{2}}{2}$$

Doug should have substituted −12 as the first value in the numerator in line 1. In line 2, Doug miscalculated the result under the radical. In line 3, he incorrectly simplified $\frac{\sqrt{80}}{4}$.

The correct solution is shown.

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(2)(-7)}}{2(2)}$$
$$x = \frac{12 \pm \sqrt{200}}{4}$$
$$x = \frac{12 \pm 10\sqrt{2}}{4}$$
$$x = \frac{6 \pm 5\sqrt{2}}{2}$$

Cumulative Review Page 265 Question 13

a) Solve $3x^2 - 6 = 0$ by taking the square root.

$$3x^2 - 6 = 0$$
$$3x^2 = 6$$
$$x = \pm\sqrt{2}$$

For $x = \sqrt{2}$:

Left Side	Right Side
$3x^2 - 6$	0
$= 3(\sqrt{2})^2 - 6$	
$= 6 - 6$	
$= 0$	

Left Side = Right Side

For $x = -\sqrt{2}$:

Left Side	Right Side
$3x^2 - 6$	0
$= 3(-\sqrt{2})^2 - 6$	
$= 6 - 6$	
$= 0$	

Left Side = Right Side

The roots are $\pm\sqrt{2}$.

b) Solve $m^2 - 15m = -26$ by factoring.

$m^2 - 15m = -26$
$m^2 - 15m + 26 = 0$
$(m - 13)(m - 2) = 0$
$m - 13 = 0$ or $m - 2 = 0$
$m = 13$ $\quad$ $m = 2$

For $m = 13$:

Left Side	Right Side
$m^2 - 15m$	-26
$= 13^2 - 15(13)$	
$= -26$	

Left Side = Right Side

For $m = 2$:

Left Side	Right Side
$m^2 - 15m$	-26
$= 2^2 - 15(2)$	
$= -26$	

Left Side = Right Side

The roots are 13 and 2.

c) Solve $s^2 - 2s - 35 = 0$ by factoring.

$s^2 - 2s - 35 = 0$
$(s - 7)(s + 5) = 0$
$s - 7 = 0$ or $s + 5 = 0$
$s = 7$ $\quad$ $s = -5$

For $s = 7$:

Left Side	Right Side
$s^2 - 2s - 35$	0
$= 7^2 - 2(7) - 35$	
$= 49 - 14 - 35$	
$= 0$	

Left Side = Right Side

For $s = -5$:

Left Side	Right Side
$s^2 - 2s - 35$	0
$= (-5)^2 - 2(-5) - 35$	
$= 25 + 10 - 35$	
$= 0$	

Left Side = Right Side

The roots are 13 and 2.

d) Solve $-16x^2 + 47x + 3 = 0$ by factoring.

$-16x^2 + 47x + 3 = 0$
$-(16x^2 - 47x - 3) = 0$
$-(16x + 1)(x - 3) = 0$
$16x + 1 = 0$ or $x - 3 = 0$
$x = -\frac{1}{16}$ $\quad$ $x = 3$

For $x = -\frac{1}{16}$:

Left Side	Right Side
$-16x^2 + 47x + 3$	0

For $x = 3$:

Left Side	Right Side
$-16x^2 + 47x + 3$	0

$= -16\left(-\frac{1}{16}\right)^2 + 47\left(-\frac{1}{16}\right) + 3$

$= -\frac{1}{16} - \frac{47}{16} + \frac{48}{16}$

$= 0$

Left Side = Right Side

$= -16(3)^2 + 47(3) + 3$

$= -144 + 141 + 3$

$= 0$

Left Side = Right Side

Cumulative Review Page 265 Question 14

a) For $x^2 - 6x + 3 = 0$, $a = 1$, $b = -6$, and $c = 3$.
$b^2 - 4ac = (-6)^2 - 4(1)(3)$
$b^2 - 4ac = 36 - 12$
$b^2 - 4ac = 24$
Since the value of the discriminant is positive, there are two distinct real roots.

b) For $x^2 + 22x + 121 = 0$, $a = 1$, $b = 22$, and $c = 121$.
$b^2 - 4ac = 22^2 - 4(1)(121)$
$b^2 - 4ac = 484 - 484$
$b^2 - 4ac = 0$
Since the value of the discriminant is zero, there is one distinct real root.

c) For $-x^2 + 3x - 5 = 0$, $a = -1$, $b = 3$, and $c = -5$.
$b^2 - 4ac = 3^2 - 4(-1)(-5)$
$b^2 - 4ac = 9 - 20$
$b^2 - 4ac = -11$
Since the value of the discriminant is negative, there are no real roots.

Cumulative Review Page 265 Question 15

a) Let x represent the side length of the bottom square. Then, $x + 1$ represents the side length of the top square. For a total area of 85 in.^{2},
$85 = x^2 + (x + 1)^2$

b) Solve by factoring.
$85 = x^2 + (x + 1)^2$
$0 = 2x^2 + 2x - 84$
$0 = 2(x^2 + x - 42)$
$0 = 2(x + 7)(x - 6)$
$x + 7 = 0$ or $x - 6 = 0$
$x = -7$ $\quad$ $x = 6$

c) The side length of the bottom of the box is 6 in. and the side length of the the top of the box is 7 in..

d) Since the side length must be positive, $x = -7$ is an extraneous root.

Unit 2 Test

Unit 2 Test Page 266 Question 1

The graph of the function that is congruent to the graph of $f(x) = x^2 + 3$ but translated vertically 2 units down is $f(x) = x^2 + 1$. Choice **A**.

Unit 2 Test Page 266 Question 2

For a quadratic function with a vertex at (–1, –2) and passing through the point (1, 6), the equation is of the form $y = a(x + 1)^2 - 2$. Use the given point to find a.

$y = a(x + 1)^2 - 2$
$6 = a(1 + 1)^2 - 2$
$8 = 4a$
$a = 2$

The equation of the quadratic function is $y = 2(x + 1)^2 - 2$. Choice **D**.

Unit 2 Test Page 266 Question 3

The graph of $y = ax^2 + q$ intersects the x-axis in two places when $a > 0$ and the vertex is below the x-axis, or $q < 0$, or when $a < 0$ and the vertex is above the x-axis, or $q > 0$. Choice **D**.

Unit 2 Test Page 266 Question 4

Change $y = 2x^2 - 8x + 2$ to vertex form.

$y = 2x^2 - 8x + 2$
$y = 2(x^2 - 4x) + 2$
$y = 2(x^2 - 4x + 4 - 4) + 2$
$y = 2[(x^2 - 4x + 4) - 4] + 2$
$y = 2(x - 2)^2 - 8 + 2$
$y = 2(x - 2)^2 - 6$

So, $p = 2$ and $q = -6$. Choice **B**.