

8.2 Solving Systems of Equations Algebraically

Example 1

Solve a System of Linear-Quadratic Equations Algebraically

a) Solve the following system of equations.

$$\begin{aligned} 1 \quad & 5x - y = 10 \quad \rightarrow \quad 10x - 2y = 20 \\ 2 \quad & x^2 + x - 2y = 0 \quad \rightarrow \quad 10x - 20 = 2y \end{aligned}$$

$$2 \quad x^2 + x = 2y$$

$$1 \quad 10x - 20 = 2y$$

elimination

$$x^2 - 9x + 20 = 0$$

$$(x-5)(x-4) = 0$$

$$x = 5 \quad x = 4$$

$$\begin{aligned} 25 - y &= 10 & 20 - y &= 10 \\ -y &= -15 & -y &= -10 \end{aligned}$$

$$y = 15 \quad y = 10$$

$$(5, 15) \quad (4, 10)$$

$$1 \quad 5x - 10 = y \quad \text{substitution}$$

$$2 \quad x^2 + x - 2(5x - 10) = 0$$

$$x^2 + x - 10x + 20 = 0$$

$$x^2 - 9x + 20 = 0$$

same

Example 2

Model a Situation With a System of Equations

Determine two integers such that the sum of the smaller number and twice the larger number is 46. Also, when the square of the smaller number is decreased by three times the larger, the result is 93. In the box below, enter the smaller number followed by the larger number and you will gain access to our site.

① $x + 2y = 46$ (y larger)

② $x^2 - 3y = 93$

② $x^2 - 3y = 93$
 $2x^2 - 6y = 186$
 $3x + 6y = 138$

 $2x^2 + 3x = 324$

$2x^2 + 3x - 324 = 0$
 $(2x + 27)(x - 12) = 0$

$x = -\frac{27}{2}$
 $x = 12$

(not integer)

$3 \pm \sqrt{9 - 4(2)(-324)}$

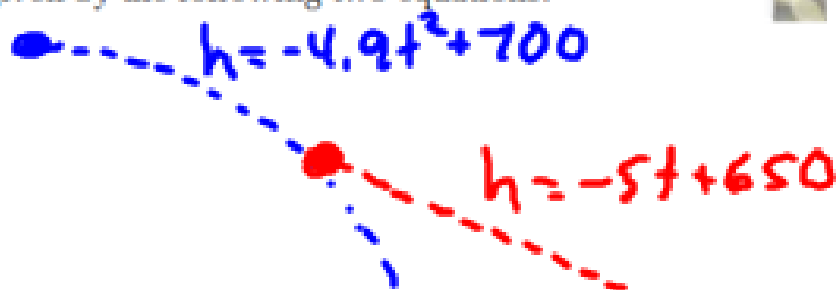
$12 + 2y = 46$
 $2y = 34$
 $y = 17$

12 and 17

Example 3

Solve a Problem Involving a Linear-Quadratic System

A Canadian cargo plane drops a crate of emergency supplies to aid-workers on the ground. The crate drops freely at first before a parachute opens to bring the crate gently to the ground. The crate's height, h , in metres, above the ground t seconds after leaving the aircraft is given by the following two equations.



b) sub $t = 3.75$ in either
 $h = 631.27 \text{ m}$

$h = -4.9t^2 + 700$ represents the height of the crate during free fall.

$h = -5t + 650$ represents the height of the crate with the parachute open.

- How long after the crate leaves the aircraft does the parachute open? Express your answer to the nearest hundredth of a second.
- What height above the ground is the crate when the parachute opens? Express your answer to the nearest metre.

$$\begin{aligned} -4.9t^2 + 700 &= -5t + 650 \\ -4.9t^2 + 5t + 50 &= 0 \end{aligned}$$

$$t = \frac{-5 \pm \sqrt{25 - 4(-4.9)(50)}}{-9.8}$$

$$t = 3.75 \text{ and } -2.72$$

(a) $t = 3.75 \text{ sec.}$

Example 4

Solve a System of Quadratic-Quadratic Equations Algebraically

a) Solve the following system of equations.

$$3x^2 - x - y - 2 = 0 \rightarrow 3x^2 - x - y - 2 = 0$$

$$6x^2 + 4x - y = 4 \rightarrow 6x^2 + 4x - y - 4 = 0$$

$$\underline{-3x^2 - 5x + 2 = 0}$$

$$3x^2 + 5x - 2 = 0$$

$$(3x - 1)(x + 2) = 0$$

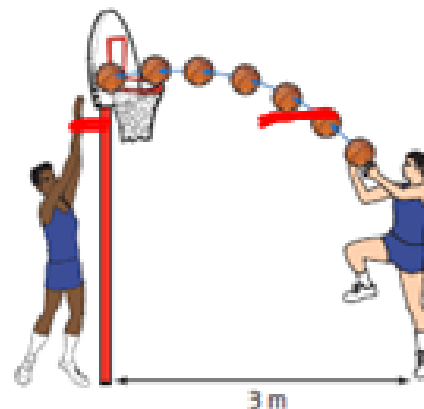
$$x = \frac{1}{3} \quad x = -2$$

$$\begin{aligned} 3(-2)^2 + 2 - y - 2 &= 0 \\ -y &= -12 \\ y &= 12 \end{aligned}$$

$$\boxed{\left(\frac{1}{3}, -2\right) \quad (-2, 12)}$$

Example 5

Solve a Problem Involving a Quadratic-Quadratic System



During a basketball game, Ben completes an impressive "alley-oop." From one side of the hoop, his teammate Luke lobs a perfect pass toward the basket. Directly across from Luke, Ben jumps up, catches the ball and tips it into the basket. The path of the ball thrown by Luke can be modelled by the equation $d^2 - 2d + 3h = 9$, where d is the horizontal distance of the ball from the centre of the hoop, in metres, and h is the height of the ball above the floor, in metres.

The path of Ben's jump can be modelled by the equation $5d^2 - 10d + h = 0$, where d is his horizontal distance from the centre of the hoop, in metres, and h is the height of his hands above the floor, in metres.

- Solve the system of equations algebraically. Give your solution to the nearest hundredth.
- Interpret your result. What assumptions are you making?

$$\begin{aligned}d^2 - 2d + 3h &= 9 \rightarrow 5d^2 - 10d + 15h = 45 \\ \underline{5d^2 - 10d + h} &= 0 \\ -14h &= -45 \\ h &= \frac{45}{14} \\ h &= \underline{\underline{3.21}}\end{aligned}$$

$$\begin{aligned}d^2 - 2d + 3h &= 9 \\ \underline{15d^2 - 30d + 3h} &= 0 \\ -14d^2 + 28d &= 9 \\ -14d^2 + 28d - 9 &= 0 \\ \downarrow \text{quad. formula} \\ d &= 1.60 \text{ and } 0.40 \\ \downarrow \\ d &= 1.6 \text{ m} \\ h &= 3.21 \text{ m} \\ \text{from passer}\end{aligned}$$

Today 3a,c,e, 4a,c,e, 5c, 6, 7, 8, 9 pgs.451-452

Tomorrow 10-13, 14c,d, 16,19,20 pgs.453-455

Tuesday: In Class Assignment on 8.1, 8.2