

7.4 Reciprocal Functions

Example 1

Compare the Graphs of a Function and Its Reciprocal

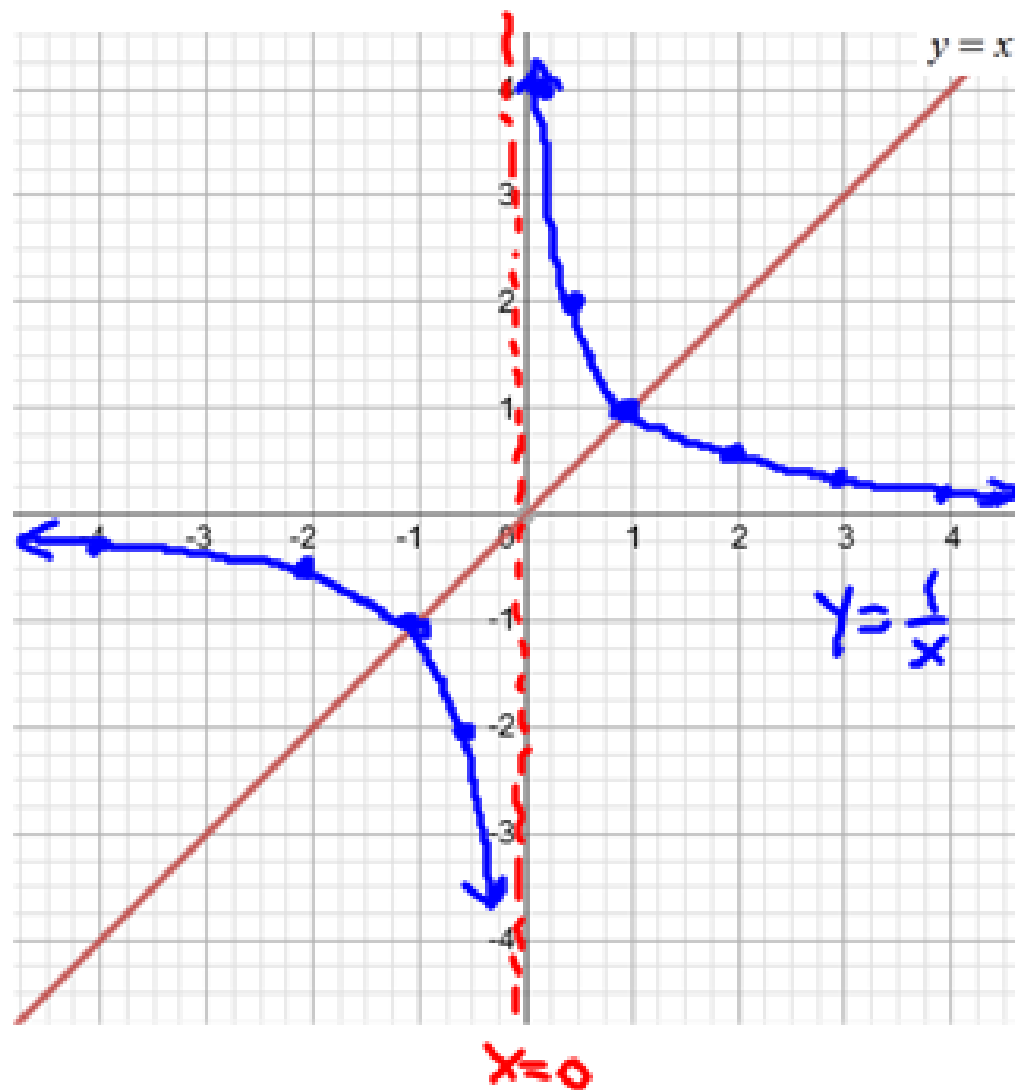
Sketch the graphs of $y = f(x)$ and its **reciprocal function** $y = \frac{1}{f(x)}$, where $f(x) = x$. Examine how the functions are related.

\downarrow
 $Y = X$
 Reciprocal
 $Y = \frac{1}{x}$

x	y = x	y = $\frac{1}{x}$
-10	-10	$-\frac{1}{10}$
-5	-5	$-\frac{1}{5}$
-2	-2	$-\frac{1}{2}$
-1	-1	-1
$-\frac{1}{2}$	$-\frac{1}{2}$	-2
$-\frac{1}{5}$	$-\frac{1}{5}$	-5
$-\frac{1}{10}$	$-\frac{1}{10}$	-10
0	0	undefined
$\frac{1}{10}$	$\frac{1}{10}$	10
$\frac{1}{5}$	$\frac{1}{5}$	5
$\frac{1}{2}$	$\frac{1}{2}$	2
1	1	1
2	2	$\frac{1}{2}$
5	5	$\frac{1}{5}$
10	10	$\frac{1}{10}$

x	y	reciprocal
-3	-3	$-\frac{1}{3}$
-2	-2	$-\frac{1}{2}$
-1	-1	-1
0	0	undefined.
1	1	1
2	2	$\frac{1}{2}$

horizontal asymptote
 $y=0$



Patterns:

when $y=1$, or $y=-1$: original/reciprocal cross/touch

when $y=0$: reciprocal has a vertical asymptote $x = \sim$

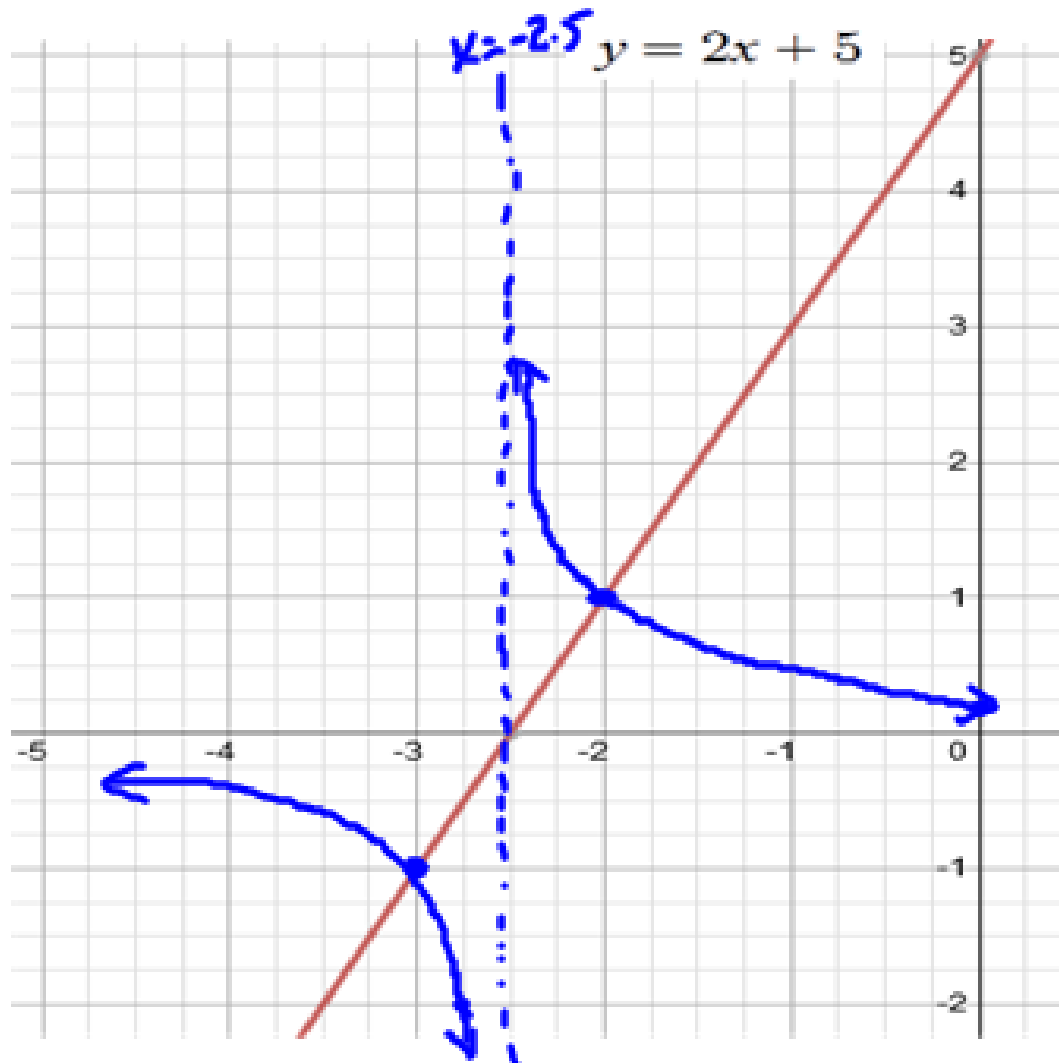
as y approaches zero: the reciprocal $\rightarrow \infty$

$y \rightarrow 0^+$	$\frac{1}{y} \rightarrow \infty$
$y \rightarrow 0^-$	$\frac{1}{y} \rightarrow -\infty$

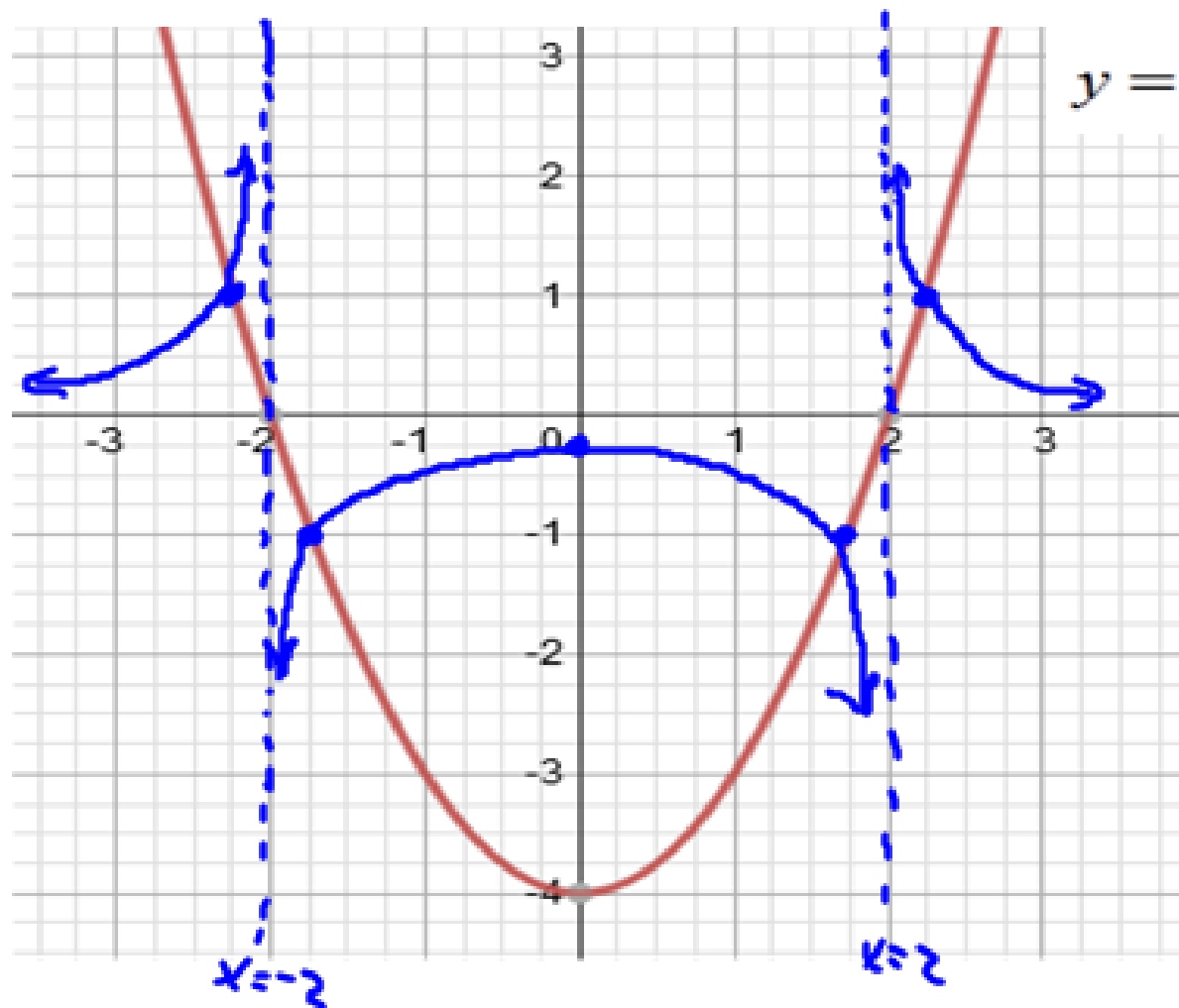
as y grows large: reciprocal value $\rightarrow 0$

as $y \rightarrow \infty$	$\frac{1}{y} \rightarrow 0^+$	} Horizontal asymptote $y=0$
$y \rightarrow -\infty$	$\frac{1}{y} \rightarrow 0^-$	

$$y = \frac{1}{2x+5}$$



original
 $0 \rightarrow VA$
 $\pm \rightarrow \pm 1$



$$y = x^2 - 4$$

$$y = \frac{1}{x^2 - 4}$$

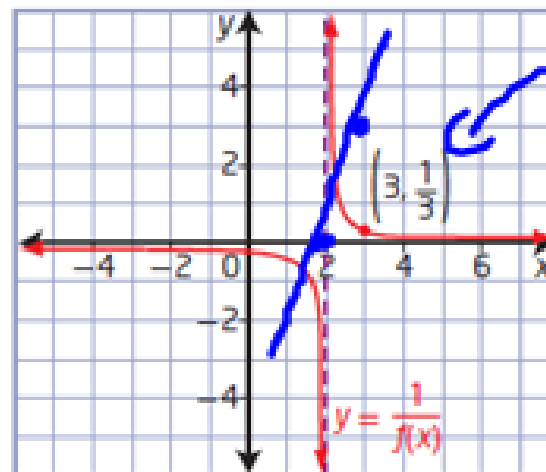
$$\lim_{x \rightarrow \infty} \frac{1}{x^2 - 4} = 0$$

Example 4

Graph $y = f(x)$ Given the Graph of $y = \frac{1}{f(x)}$

The graph of a reciprocal function of the form $y = \frac{1}{ax + b}$, where a and b are non-zero constants, is shown.

- Sketch the graph of the original function, $y = f(x)$.
- Determine the original function, $y = f(x)$.



$3, \frac{1}{3}$ on reciprocal
means
 $3, 3$ on original

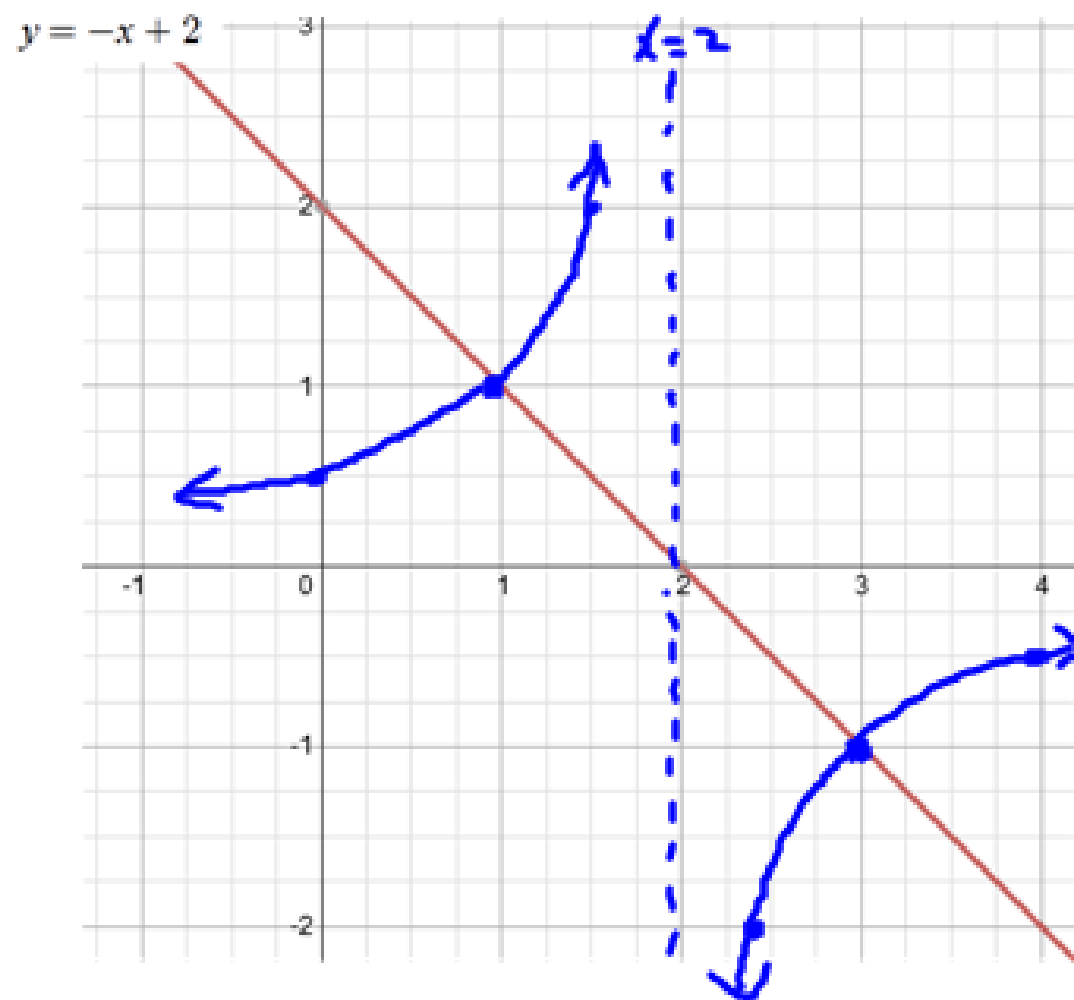
$(2, 0)$ $(3, 3)$
slope $\frac{3}{1} = 3$

$\therefore y = 3x + b$

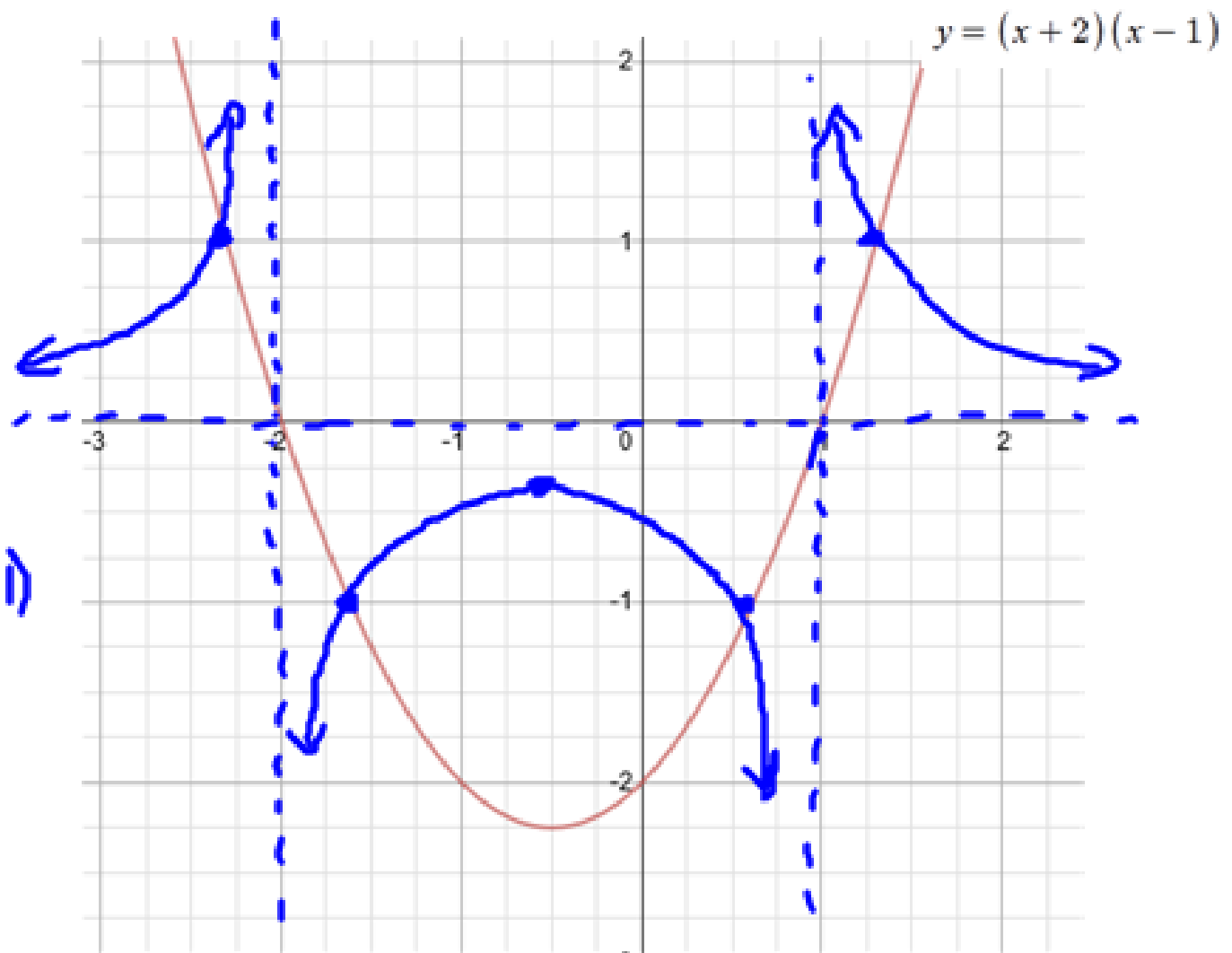
$(3, 3)$ $3 = 9 + b$ $b = -6$ $y = 3x - 6$

VA at $x = 2$

original $(2, 0)$



$$Y = \frac{1}{(x+2)(x-1)}$$



Today: 7.4 1-8 (7a,7c,8a,8c) pgs.403-404

Tomorrow: 9-16 pgs.405-407