

6.4 Rational Equations

Steps:

- get common denominators
- multiply by common denominators to "drop" denominators
- solve
- beware of restricted (non-admissible) values

Example 1

Solve a Rational Equation

Solve the following equation.

$$\frac{2}{z^2 - 4} + \frac{10}{6z + 12} = \frac{1}{z - 2}$$

$$\frac{2}{(z+2)(z-2)} + \frac{10}{6(z+2)} = \frac{1}{z-2}$$

$$\frac{12 \cancel{6(z+2)(z-2)}}{6 \cancel{(z+2)} \cancel{(z-2)}} + \frac{10(z-2) \cancel{6(z+2)(z-2)}}{6 \cancel{(z+2)} \cancel{(z-2)}} = \frac{6(z+2) \cancel{6(z+2)(z-2)}}{6 \cancel{(z+2)} \cancel{(z-2)}}$$

$$z \neq 2, -2$$

$$12 + 10(z-2) = 6(z+2)$$

$$12 + 10z - 20 = 6z + 12$$

$$\rightarrow 4z = 20$$
$$\boxed{z = 5}$$

Example 2

Solve a Rational Equation

$$\frac{4k-1}{k+2} - \frac{k+1}{k-2} = \frac{k^2-4k+24}{k^2-4}$$

$$\frac{4k-1}{k+2} - \frac{k+1}{k-2} = \frac{k^2-4k+24}{(k-2)(k+2)}$$

$$\frac{(4k-1)(k-2)}{(k+2)(k-2)} - \frac{(k+1)(k+2)}{(k+2)(k-2)} = \frac{k^2-4k+24}{(k+2)(k-2)}$$

$$4k^2-9k+2 - k^2-3k-2 = k^2-4k+24$$

$$2k^2-8k-24=0$$

$$k^2-4k-12=0$$

$$(k-6)(k+2)=0$$

$$k=6 \quad k=-2$$

but $k \neq \pm 2$

$$\boxed{k=6}$$

Example 3

Use a Rational Equation to Solve a Problem

Two friends share a paper route. Sheena can deliver the papers in 40 min. Jeff can cover the same route in 50 min. How long, to the nearest minute, does the paper route take if they work together?

The textbook is big on making these charts:

Make a table to organize the information.

	Time to Deliver Papers (min)	Fraction of Work Done in 1 min	Fraction of Work Done in t minutes
Sheena	40	$\frac{1}{40}$	$(\frac{1}{40})(t)$ or $\frac{t}{40}$
Jeff	50	$\frac{1}{50}$	$\frac{t}{50}$
Together	t	$\frac{1}{t}$	$\frac{t}{t}$ or 1

$$\frac{t}{40} + \frac{t}{50} = \frac{1}{1}$$

$$\frac{5t}{200} + \frac{4t}{200} = \frac{200}{200}$$

$$5t + 4t = 200$$

$$9t = 200$$

$$t = \frac{200}{9} \text{ min}$$

$$t \approx 22.2 \text{ min}$$

Example 4

Use a Rational Equation to Solve a Problem

The Northern Manitoba Trapper's Festival, held in The Pas, originated in 1916. A championship dog race has always been a significant part of the festivities. In the early days, the race was non-stop from The Pas to Flin Flon and back.

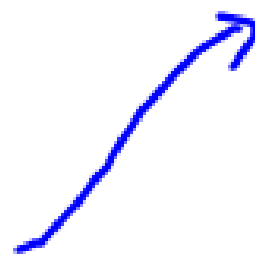
In one particular race, the total distance was 140 mi. Conditions were excellent on the way to Flin Flon. However, bad weather caused the winner's average speed to decrease by 6 mph on the return trip. The total time for the trip was $8\frac{1}{2}$ h. What was the winning dog team's average speed on the way to Flin Flon?



speed = $\frac{\text{distance}}{\text{time}}$

$$V = \frac{d}{t}$$

$$t = \frac{d}{V}$$



to FlinFlon back total

$$\boxed{\frac{70}{V} + \frac{70}{V-6} = 8.5}$$

$$\frac{70(V-6)}{V(V-6)} + \frac{70(V)}{(V-6)V} = \frac{8.5V(V-6)}{V(V-6)}$$

$$70(V-6) + 70V = 8.5V(V-6)$$

$$70V - 420 + 70V = 8.5V^2 - 51V$$

$$0 = 8.5V^2 - 191V + 420$$

$$0 = 17V^2 - 382V + 840$$

$$0 = (17V - 42)(V - 20)$$

$$V = \frac{42}{17} \quad V = 20$$

$$V = \frac{42}{17} \text{ or } V = 20$$

$$2.47$$

↑
V-6 is
negative

$$\boxed{20 \text{ km/h}}$$

$$\boxed{1-8pg 348}$$