6.3

3 a) $\frac{1}{(x-3)(x+1)} - \frac{4(x-3)}{(x-3)(x+1)} = \frac{1-4x+12}{(x-3)(x+1)} = \frac{13-4x}{(x-3)(x+1)}$ $x \neq 3, -1$

b) $\frac{x-5}{(x+10)(x-2)} + \frac{2x+1}{(x+2)(x-2)} = \frac{(x-5)(x+2) + (2x+1)(x+10)}{(x+10)(x-2)(x+2)}$

$= \frac{(x^2-3x-10) + (2x^2+21x+10)}{(x+10)(x-2)(x+2)}$

$= \frac{3x^2+18x}{(x+10)(x-2)(x+2)}$

$= \frac{3x(x+6)}{(x+10)(x-2)(x+2)}$ $x \neq \pm 2, -10$

4 a) LCD is 12

b) LCD is $10a^2y^2$

c) LCD is $(3+x)(3-x)$ or $9-x^2$

5. a) $\frac{5}{15a} + \frac{6}{15a} = \frac{11}{15a}$ $a \neq 0$

b) $\frac{9}{6x} + \frac{x}{6x} = \frac{9+x}{6x}$ $x \neq 0$

c) $\frac{20x}{5x} - \frac{6}{5x} = \frac{20x-6}{5x}$ $x \neq 0$

d) $\frac{4z^2}{xyz} - \frac{9x^2}{xyz} = \frac{4z^2-9x^2}{xyz}$ $x, y, z, \neq 0$

e) $\frac{12st}{30t^3} + \frac{3t^2}{30t^3} - \frac{12}{30t^3} = \frac{12st+3t^2-12}{30t^3} = \frac{4st+t^2-4}{10t^3}$ $t \neq 0$

f) $\frac{66xy^2}{a^2b^2y} - \frac{2ax}{a^2b^2y} + \frac{a^2b^2y}{a^2b^2y} = \frac{66xy^2-2ax+a^2b^2y}{a^2b^2y}$ $a, b, y \neq 0$

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6. a) $\frac{8}{(x+2)(x-2)} - \frac{5(x-2)}{(x+2)(x-2)} = \frac{8-5x+10}{(x+2)(x-2)} = \frac{18-5x}{(x+2)(x-2)}$ $x \neq \pm 2$

b) $\frac{1}{(x-4)(x+3)} + \frac{3(x-4)}{(x-4)(x+3)} = \frac{1+3x-12}{(x-4)(x+3)} = \frac{3x-11}{(x-4)(x+3)}$ $x \neq 4, -3$

c) $\frac{3x(x-2)}{(x+2)(x-2)} - \frac{x(x+2)}{(x+2)(x-2)} = \frac{(3x^2-6x)-(x^2+2x)}{(x+2)(x-2)} = \frac{2x^2-8x}{(x+2)(x-2)}$

$\frac{2x(x-4)}{(x+2)(x-2)}$ $x \neq \pm 2$

d) $\frac{5y}{y(y+1)} - \frac{y+1}{y(y+1)} - \frac{y-4}{y(y+1)} = \frac{5y-y-1-y+4}{y(y+1)} = \frac{3y+3}{y(y+1)}$

$\frac{3(y+1)}{y(y+1)} = \frac{3}{y}$ $y \neq 0, -1$

e) $\frac{2h(h+3)}{(h+3)(h-3)(h+3)} + \frac{h(h-3)}{(h+3)(h-3)(h+3)} - \frac{3(h+3)(h+3)}{(h+3)(h+3)(h-3)}$

$\frac{2h^2+6h+h^2-3h-3h^2-18h-27}{(h+3)(h+3)(h-3)} = \frac{-15h-27}{(h+3)(h+3)(h-3)}$

$= \frac{-3(5h+9)}{(h+3)(h+3)(h-3)}$ $h \neq \pm 3$

f) $\frac{2x(x-1)}{x(x+3)(x-2)(x-1)} + \frac{3(x-2)}{x(x+3)(x-1)(x-2)} = \frac{2x^2-2x+3x-6}{x(x-1)(x-2)(x+3)}$

$\frac{2x^2+x-6}{}$ $\frac{(2x-3)(x+2)}{x(x-1)(x-2)(x+3)}$ $x \neq 0, 1, 2, -3$

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7a) $\frac{3(x+5)}{(x+5)(x-5)} + \frac{(2x+1)(2x-1)}{(2x-1)(x+5)}$ $\quad x \neq \frac{1}{2}, \pm 5$

$\frac{3}{x-5} + \frac{2x+1}{x+5}$

$\frac{3(x+5) + (2x+1)(x-5)}{(x-5)(x+5)} = \frac{3x+15+2x^2-9x-5}{(x-5)(x+5)}$

$= \frac{2x^2-6x+10}{(x-5)(x+5)} = \frac{2(x^2-3x+5)}{(x-5)(x+5)}$

$x \neq \frac{1}{2}, \pm 5$

b) $\frac{2x}{x(x+3)(x-2)} - \frac{x-8}{(x-8)(x+3)}$

$\frac{2}{(x+3)(x-2)} - \frac{1(x-2)}{(x+3)(x-2)} = \frac{2-x+2}{(x+3)(x-2)} = \frac{4-x}{(x+3)(x-2)}$ $\quad x \neq 0, 2, -3, 8$

c) $\frac{n+3}{(n-3)(n-2)} \cdot \frac{(n-4)}{(n-4)} + \frac{6(n-2)}{(n-3)(n-4)(n-2)} = \frac{n^2-n-12+6n-12}{(n-2)(n-3)(n-4)}$

$= \frac{n^2+5n-24}{(n-2)(n-3)(n-4)} = \frac{(n+8)(n-3)}{(n-2)(n-3)(n-4)}$

$= \frac{n+8}{(n-2)(n-4)}$ $\quad n \neq 2, 3, 4$

d) $\frac{2w(w+4)}{(w+2)(w+3)(w+4)} - \frac{(w-6)(w+3)}{(w+2)(w+4)(w+3)}$ $\quad \frac{(2w^2+8w) - (w^2-3w-18)}{(w+2)(w+3)(w+4)}$

$\frac{w^2+11w+18}{(w+2)(w+3)(w+4)} = \frac{(w+2)(w+9)}{(w+2)(w+3)(w+4)} = \frac{w+9}{(w+3)(w+4)}$ $\quad w \neq -2, -3, -4$

8. Sign $\frac{6x+12+4-7x+14}{(x-2)(x+2)} = \frac{-x+30}{(x-2)(x+2)}$ $\quad x \neq \pm 2$

9. $\frac{-x+5}{(x-5)(x+5)} = \frac{-(x-5)}{(x-5)(x+5)} = -\frac{1}{x+5}$ $\quad x \neq \pm 5$

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10 a) $\dfrac{\frac{2x}{x}-\frac{6}{x}}{\frac{x^2}{x^2}-\frac{9}{x^2}}$ $\qquad \dfrac{2x-6}{x} \div \dfrac{x^2-9}{x^2}$ $\qquad \dfrac{2x-6}{x} \cdot \dfrac{x^2}{x^2-9}$

$= \dfrac{2(x-3)}{x} \cdot \dfrac{x \cdot x}{(x+3)(x-3)}$

$= \dfrac{2x}{x+3} \qquad x \neq 0, \pm 3$

b) $\dfrac{\frac{3t}{2t}+\frac{6}{2t}}{\frac{t^2}{t(t+6)}-\frac{t+6}{t(t+6)}}$ $\qquad \dfrac{3t+6}{2t} \div \dfrac{t^2-t-6}{t(t+6)}$ $\qquad \dfrac{3(t+2)}{2t} \cdot \dfrac{t(t+6)}{(t-3)(t+2)}$

$= \dfrac{3(t+6)}{2(t-3)} \qquad t \neq 0, 3, -2, -6$

c) $\dfrac{\frac{3(2m+3)}{m(2m+3)}-\frac{3m}{m(2m+3)}}{\frac{3(2m+3)}{m^2(2m+3)}+\frac{m^2}{m^2(2m+3)}}$ $\qquad \dfrac{3m+9}{m(2m+3)} \div \dfrac{m^2+6m+9}{m^2(2m+3)}$

$= \dfrac{3(m+3)}{m(2m+3)} \cdot \dfrac{m^2(2m+3)}{(m+3)(m+3)}$

$= \dfrac{3m}{m+3} \qquad m \neq 0, -\frac{3}{2}, -3$

d) $\dfrac{\frac{x-4}{(x+4)(x-4)}+\frac{x+4}{(x+4)(x-4)}}{\frac{x}{(x+4)(x-4)}+\frac{x-4}{(x+4)(x-4)}}$ $\qquad \dfrac{2x}{x^2-16} \div \dfrac{2x-4}{x^2-16}$

$= \dfrac{2x}{x^2-16} \cdot \dfrac{x^2-16}{2(x-2)}$

$= \dfrac{x}{x-2} \qquad x \neq 2, \pm 4$

11 a) $\dfrac{A}{B}+\dfrac{C}{D}$ $\qquad \dfrac{\frac{AD}{B}+\frac{BC}{B}}{D} = \dfrac{AD+BC}{B} \div D$

$= \dfrac{AD+BC}{B} \cdot \dfrac{1}{D}$

$\dfrac{AD+CB}{BD}$ $\longleftarrow$ equal $\longrightarrow$ $= \dfrac{AD+BC}{BD}$

Hilary

11b) $\left[\frac{\left(\frac{AB}{D}+\frac{C}{1}\right)D}{F}+E\right]F$ → remove inner brackets. $= \left[\frac{AB+CD}{F}+E\right]F$ → remove brackets. $= AB+CD+EF$ ✓

12. $\left(\frac{x}{2}\right)^2+\left(\frac{x-1}{4}\right)^2=h^2$

$\frac{x^2}{4}+\frac{x^2-2x+1}{16}=h^2$

$\frac{4x^2}{16}+\frac{x^2-2x+1}{16}=h^2$

$h^2=\frac{5x^2-2x+1}{16}$

$h=\frac{\sqrt{5x^2-2x+1}}{4}$

13a) expected weeks $=\frac{200}{m}$ actual weeks $=\frac{200}{m+4}$

b) $\frac{200}{m}-\frac{200}{m+4}$ c) $\frac{200(m+4)}{m(m+4)}-\frac{200m}{m(m+4)}=\frac{800}{m(m+4)}$ $m\neq 0,-4$

14a) $\frac{200}{n}$ b) $\frac{200}{n}+\frac{500}{n}+\frac{1000}{n}$ c) $\frac{1700}{n}$

d) $\frac{200}{n}+\frac{500}{n-5}+\frac{1000}{n-10}-\frac{1700}{n}$ (new − old) $-\frac{1500}{n}+\frac{500}{n-5}+\frac{1000}{n-10}$

$\frac{-1500(n-5)(n-10)+500n(n-10)+1000(n)(n-5)}{n(n-5)(n-10)}$

$\frac{-1500(n^2-15n+50)+500n^2-5000n+1000n^2-5000n}{n(n-5)(n-10)}$

$\frac{-1500n^2+22500n-75000+1500n^2-10000n}{n(n-5)(n-10)}$

$\frac{12500n-75000}{n(n-5)(n-10)}$ $n\neq 0,5,10$ or $\frac{12500(n-6)}{n(n-5)(n-10)}$

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15 a) $\frac{x-2}{x+5} + \frac{(x-3)(x+1)}{(x-3)(x+2)} \cdot \frac{x(x+2)}{x(x-4)}$

$\frac{x-2}{x+5} + \frac{x+1}{x-4}$

$\frac{(x-2)(x-4)}{(x+5)(x-4)} + \frac{(x+1)(x+5)}{(x-4)(x+5)} = \frac{(x^2-6x+8)+(x^2+6x+5)}{(x-4)(x+5)}$

$= \frac{2x^2+13}{(x-4)(x+5)}$ $x \neq -5, -2, 0, 3, 4$

b) $\frac{x(2x-1)}{x(x+3)} \cdot \frac{(x-4)(x+3)}{(2x-1)(x-1)} - \frac{x-1}{x+2}$

$\frac{x-4}{x-1} - \frac{x-1}{x+2}$ $\frac{(x-4)(x+2)-(x-1)(x-1)}{(x-1)(x+2)}$

$= \frac{x^2-2x-8-x^2+2x-1}{(x-1)(x+2)}$

$= \frac{-9}{(x-1)(x+2)}$ $x \neq 0, -3, \frac{1}{2}, 1, -2$

c) $\frac{x-2}{x+5} - \frac{(x-3)(x+1)}{(x-3)(x+2)} \cdot \frac{x(x+2)}{x(x-4)}$

$\frac{x-2}{x+5} - \frac{x+1}{x-4} = \frac{(x^2-6x+8)-(x^2+6x+5)}{(x+5)(x-4)}$

$= \frac{-12x+3}{(x+5)(x-4)}$ or $\frac{-3(4x-1)}{(x+5)(x-4)}$ $x \neq -5, -2, 0, 3, 4$

d) $\frac{x+1}{x+6} - \frac{(x+2)(x-2)}{x(x+2)} \cdot \frac{x(2x+1)}{(2x+1)(x+3)}$

$\frac{x+1}{x+6} - \frac{x-2}{x+3} = \frac{(x^2+4x+3)-(x^2+4x-12)}{(x+6)(x+3)}$

$= \frac{15}{(x+6)(x+3)}$ $x \neq 0, -2, -3, -6, -\frac{1}{2}$

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18 a) $\frac{a^2}{ab} - \frac{b^2}{ab} = \frac{a^2-b^2}{ab}$ * common denominator

b) $\frac{c(a+b)}{c(1+d)} = \frac{a+b}{1+d}$ c(1): students often "lose" the 1

c) $\frac{a-(6-b)}{4} = \frac{a-6+b}{4}$ * brackets help with signs.

d) $\frac{1}{1-\frac{a}{b}} = \frac{1}{\frac{b}{b}-\frac{a}{b}} = \frac{1}{\frac{b-a}{b}} = \frac{b}{b-a}$ reciprocal of entire denominator

e) $\frac{1}{a-b} = \frac{-1}{-a+b}$ or $\frac{-1}{b-a}$ multiply entire denominator by -1

19 a) agree

b) not necessarily. Common factors that were removed might have meant a different question

$3 - \frac{7}{x} \leftarrow \frac{3x}{x} - \frac{7}{x} \leftarrow \frac{3x-7}{x}$

but original could have been $\frac{3(x+1)}{(x+1)} - \frac{7(x+2)}{x(x+2)}$

21. Given $\frac{a}{c} = \frac{b}{d}$ Prove $\frac{a}{c} = \frac{a-b}{c-d}$

↓ cross multiply

$ac - ad = ac - bc$

$bc = ad$

↓ multiply by $\frac{1}{cd}$

$\frac{bc}{cd} = \frac{ad}{cd}$

$\frac{b}{d} = \frac{a}{c}$ ✓

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23. $\left[\dfrac{p(q-x)(r-x)+q(p-x)(r-x)+r(q-x)(p-x)}{(p-x)(q-x)(r-x)}\right]$

$-\left[\dfrac{x(q-x)(r-x)+x(p-x)(r-x)+x(q-x)(p-x)}{(p-x)(q-x)(r-x)}\right]$

$= \dfrac{(p-x)(q-x)(r-x)+(q-x)(p-x)(r-x)+(r-x)(q-x)(p-x)}{(p-x)(q-x)(r-x)}$

$= \dfrac{3(p-x)(q-x)(r-x)}{(p-x)(q-x)(r-x)}$

$= 3 \qquad x \neq p, q, r$

26. $\dfrac{1}{a}+\dfrac{1}{b} = \dfrac{1}{\frac{ab}{a+b}}$ by her claim

$= \dfrac{a+b}{ab}$

$\dfrac{1}{a}+\dfrac{1}{b} = \dfrac{b}{ab}+\dfrac{a}{ab} = \dfrac{a+b}{ab}$ ✓ same result

25a) $\frac{1}{2}, \frac{1}{4} \qquad \frac{4}{8}, \frac{2}{8}$ so $\frac{3}{8}$ is halfway.

midpt $\dfrac{\frac{1}{2}+\frac{1}{4}}{2} \qquad \dfrac{\frac{2}{4}+\frac{1}{4}}{2} \qquad \frac{3}{4} \div 2 \qquad \frac{3}{4} \cdot \frac{1}{2} = \frac{3}{8}$ ✓

b) $\dfrac{\frac{3}{a}+\frac{7}{2a}}{2} \qquad \dfrac{\frac{6}{2a}+\frac{7}{2a}}{2} \qquad \frac{13}{2a} \div 2 \qquad \frac{13}{2a} \cdot \frac{1}{2} = \frac{13}{4a}$