4.3 Solving Quadratics by Completing the Square

ex1 Solve $2x^2+3x-7=0$ factoring → product -14, sum 3 (not possible)

divide by coeff of x^2: $x^2+\frac{3}{2}x-\frac{7}{2}=0$

move constant: $x^2+\frac{3}{2}x=\frac{7}{2}$

] interchangeable and "if necessary"

complete square: $x^2+\frac{3}{2}x+\frac{9}{16}=\frac{7}{2}+\frac{9}{16}$

write as square: $(x+\frac{3}{4})^2=\frac{65}{16}$ * if negative: no real roots

square root: $x+\frac{3}{4}=\pm\sqrt{\frac{65}{16}}$

$x=-\frac{3}{4}\pm\frac{\sqrt{65}}{4}$ $\quad x=\frac{-3\pm\sqrt{65}}{4}$

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1a) x^2+x+c coeff of x: $1 \xrightarrow{\text{half}} \frac{1}{2} \xrightarrow{\text{square}} \frac{1}{4}$

$x^2+x+\frac{1}{4}$

$(x+\frac{1}{2})^2$

b) x^2-5x+c coeff of x: $-5 \to -\frac{5}{2} \to \frac{25}{4}$

$x^2-5x+\frac{25}{4}$

$(x+\frac{5}{2})^2$

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2a) $2x^2+8x+4=0$

$x^2+4x+2=0$

$x^2+4x=-2$

$x^2+4x+4=-2+4$

$(x+2)^2=2$

b) $-3x^2-12x+5=0$

$x^2+4x-\frac{5}{3}=0$

$x^2+4x=\frac{5}{3}$

$x^2+4x+4=\frac{5}{3}+4$

$(x+2)^2=\frac{17}{3}$

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7a) $x^2-8x-4=0$

$x^2-8x=4$

$x^2-8x+16=4+16$

$(x-4)^2=20$

$x-4=\pm\sqrt{20}$

$x=4\pm2\sqrt{5}$ (exact)

$x=8.5, -0.5$

b) $-3x^2+4x+5=0$

$-3x^2+4x=-5$

$x^2-\frac{4}{3}x=\frac{5}{3}$

$x^2-\frac{4}{3}x+\frac{4}{9}=\frac{5}{3}+\frac{4}{9}$

$(x-\frac{2}{3})^2=\frac{19}{9}$

$x=\frac{2}{3}\pm\sqrt{19/9}$

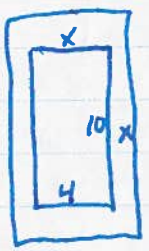
$x=\frac{2\pm\sqrt{19}}{3}$ (exact)

$x=2.1, -0.8$

finish 1,2,3,5,6,7 and do 8,9,10 pgs 240-241 (day 1)
(day 2) go over these, do 11-17

#8

a)



old area = 400

new area = 800 b) $(4+2x)(10+2x)=800$

$4x^2+28x+40=800$

$4x^2+28x=760$

$x^2+7x=190$

$x^2+7x+\frac{49}{4}=190+\frac{49}{4}$

$(x+\frac{7}{2})^2=\frac{809}{4}$

$x+\frac{7}{2}=\pm\frac{\sqrt{809}}{2}$

$x=-\frac{7}{2}\pm\frac{\sqrt{809}}{2}$

$x=-7\pm\frac{\sqrt{809}}{2}$

$x=1.2, -8.2$

dimension

6.4 x 12.4 feet

9a) $-0.02d^2+0.4d+1=0$

$d^2-20d-50=0$

$d^2-20d=50$

$d^2-20d+100=150$

$(d-10)^2=150$

$d-10=\pm\sqrt{150}$

$d=10\pm\sqrt{150}$

$d=22.2, -2.2$ $d=22.2$ m

Hilary

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#10 $h = -0.01d^2 + 2d + 1$

$0 = -0.01d^2 + 2d + 1$

$0 = d^2 - 200d \mp 100$

$d^2 - 200d = 100$

$d^2 - 200d + 10000 = 10100$

$(d-100)^2 = 10100$

$d - 100 = \sqrt{10100}$

$d = 100 \pm \sqrt{10100}$ $\quad d = 200.5, -0.5$ $\quad \therefore 200.5\text{m}$

#11 $(12-2x)(12-4x) = 54$

$8x^2 - 72x + 144 = 54$

$8x^2 - 72x + 90 = 0$

$4x^2 - 36x + 45 = 0$

$4x^2 - 36x = -45$

$x^2 - 9x = -\frac{45}{4}$

$x^2 - 9x + \frac{81}{4} = -\frac{45}{4} + \frac{81}{4}$

$(x - \frac{9}{2})^2 = 9$

$x - \frac{9}{2} = \pm 3$

$x = \frac{9}{2} \pm 3$ $\quad x = \frac{15}{2}, \frac{3}{2}$ $\quad x = 1.5$

↑ too big

$12 - 2x$ is negative

$\therefore$ 9in x 6in

#12 $0 = -0.04x^2 + 2x + 8$

$0 = x^2 - 50x \mp 200$

$x^2 - 50x = 200$

$x^2 - 50x + 625 = 825$

$(x-25)^2 = 825$

$x - 25 = \pm\sqrt{825}$

$x = 25 \pm \sqrt{825}$

$x = -3.7, 53.7$ $\quad x = 53.7\text{m}$

Hilbert

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#13 a) $(x-\sqrt{7})(x+\sqrt{7})=0$ $\quad x^2-7=0$ $\quad y=x^2-7$

b) $[x-(1+\sqrt{3})][x+(1+\sqrt{3})]$ $\quad x^2-(4+2\sqrt{3})$ awkward.

$x=1\pm\sqrt{3}$

$x-1=\pm\sqrt{3}$

$x^2-2x+1=3$ $\quad x^2-2x-2=0$ $\quad y=x^2-2x-2$

c) $x=\frac{5\pm\sqrt{11}}{2}$

$x-\frac{5}{2}=\pm\frac{\sqrt{11}}{2}$

$2x-5=\pm\sqrt{11}$

$4x^2-20x+25=11$ $\quad 4x^2-20x+14=0$

$2x^2-10x+7=0$ $\quad y=2x^2-10x+7$

#14 a) $x^2+2x+1=k+1$

$(x+1)^2=k+1$

$x+1=\pm\sqrt{k+1}$

$x=-1\pm\sqrt{k+1}$

b) $kx^2-2x=k$

$x^2-\frac{2}{k}x=1$

$x^2-\frac{2}{k}x+\frac{1}{k^2}=1+\frac{1}{k^2}$

$(x-\frac{1}{k})^2=\frac{k^2+1}{k^2}$

$x-\frac{1}{k}=\pm\sqrt{\frac{k^2+1}{k}}$

$x=\frac{1\pm\sqrt{k^2+1}}{k}$

c) $x^2-kx=1$

$x^2-kx+\frac{k^2}{4}=1+\frac{k^2}{4}$

$(x-\frac{k}{2})^2=\frac{k^2+4}{4}$

$x-\frac{k}{2}=\pm\sqrt{\frac{k^2+4}{4}}$

$x=\frac{k}{2}\pm\frac{\sqrt{k^2+4}}{2}$

$x=\frac{k\pm\sqrt{k^2+4}}{2}$

** #15 $ax^2+bx+c=0$ (ANY QUADRATIC)

$ax^2+bx=-c$

$x^2+\frac{b}{a}x=-\frac{c}{a}$

$x^2+\frac{b}{a}x+\frac{b^2}{4a^2}=-\frac{c}{a}+\frac{b^2}{4a^2}$

$(x+\frac{b}{2a})^2=\frac{b^2-4ac}{4a^2}$

$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$

$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Milroy

#16 $S_n = \frac{n}{2}(2t_1 + (n-1)d)$

$3870 = \frac{n}{2}(12 + 4(n-1))$

$3870 = \frac{n}{2}(12 + 4n - 4)$

$3870 = \frac{n}{2}(4n + 8)$

$3870 = n(2n+4)$

$3870 = 2n^2 + 4n$

$1935 = n^2 + 2n$

$1936 = n^2 + 2n + 1$

$1936 = (n+1)^2$

$44 = n+1$

$n = 43$

b) $1 + 2 + 3 + \ldots = 780$

$780 = \frac{n}{2}(2 + (n-1))$

$780 = \frac{n}{2}(n+1)$

$1560 = n(n+1)$

$1560 = n^2 + n$

$1560\frac{1}{4} = n^2 + n + \frac{1}{4}$

$\frac{6241}{4} = (n + \frac{1}{2})^2$

$\frac{79}{2} = n + \frac{1}{2}$

$\frac{78}{2} = n$

$n = 39$

#17 $12^2 = 4^2 + x^2 - 2(4)(x)\cos 60$

$144 = 16 + x^2 - 8x(\frac{1}{2})$

$0 = x^2 - 4x - 128$

$128 = x^2 - 4x$

$132 = x^2 - 4x + 4$

$132 = (x-2)^2$

$\sqrt{132} = x - 2$

$2 + \sqrt{132} = x$

$x = 2 + 2\sqrt{33}$ (exact)

$x = 13.5\text{ m}$