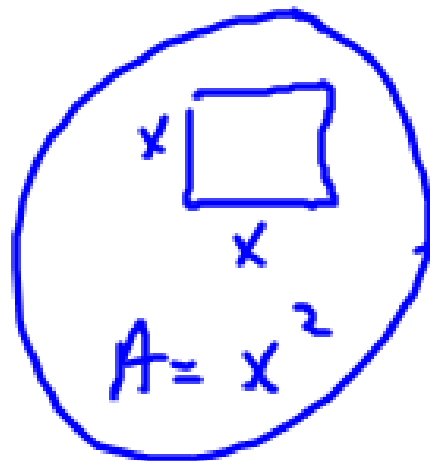
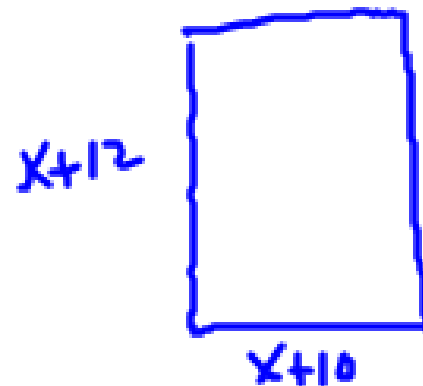


15. The area of a square is tripled by adding 10 cm to one dimension and 12 cm to the other. Determine the side length of the square.



triple Area  $3x^2$



$$A = 3x^2 \text{ AND } A = (x+10)(x+12)$$

$$3x^2 = (x+10)(x+12)$$

$$3x^2 = x^2 + 22x + 120$$

$$2x^2 - 22x - 120 = 0$$

$$x^2 - 11x - 60 = 0$$

$$(x-15)(x+4) = 0$$

$$\downarrow \quad \downarrow$$
$$x=15 \quad x=-4$$

$$\therefore x = 15 \text{ cm}$$

check

|                    |                               |
|--------------------|-------------------------------|
| $15 \times 15$     | $27 \times 25$                |
| $225 \text{ cm}^2$ | $675 \text{ cm}^2 \checkmark$ |

16. Ted popped a baseball straight up with an initial upward velocity of 48 ft/s. The height,  $h$ , in feet, of the ball above the ground is modelled by the function  $h(t) = 3 + 48t - 16t^2$ . How long was the ball in the air if the catcher catches the ball 3 ft above the ground? Is your answer reasonable in this situation? Explain.

$$3 = 3 + 48t - 16t^2$$

$$16t^2 - 48t = 0$$

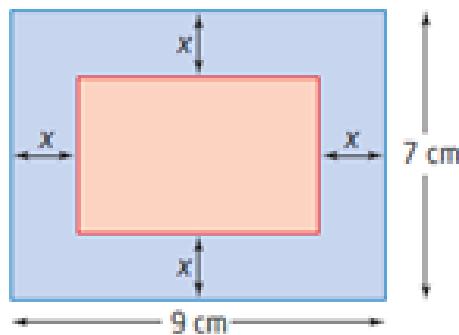
$$16t(t - 3) = 0$$

$$\begin{array}{cc} \downarrow & \downarrow \\ 16t = 0 & t - 3 = 0 \end{array}$$

$$t = 0 \quad t = 3$$

3 seconds

17. A rectangle with area of  $35 \text{ cm}^2$  is formed by cutting off strips of equal width from a rectangular piece of paper.



- What is the width of each strip?
- What are the dimensions of the new rectangle?

big rectangle  $9 \times 7$

little rectangle  $(9-2x)(7-2x)$

$$(9-2x)(7-2x) = 35$$

$$4x^2 - 32x + 63 = 35$$

$$4x^2 - 32x + 28 = 0$$

$$x^2 - 8x + 7 = 0$$

$$(x-1)(x-7) = 0$$

$$x=1 \quad x=7$$

↑ you can't cut 7cm from

a) 1cm      7cm

b) 5cm  $\times$  7cm

18. Without factoring, state if the binomial is a factor of the trinomial. Explain why or why not.

a)  $x^2 - 5x - 36$ ,  $x - 5$

b)  $x^2 - 2x - 15$ ,  $x + 3$

c)  $6x^2 + 11x + 4$ ,  $4x + 1$

d)  $4x^2 + 4x - 3$ ,  $2x - 1$

→ if  $x - 5$  is a factor, 5 is a root

$$5^2 - 5(5) - 36$$

$$25 - 25 - 36$$

$-36 \rightarrow$  not zero, 5 is not a root

∴  $x - 5$  is not a factor.

b) check -3

$$(-3)^2 - 2(-3) - 15$$

$$9 + 6 - 15$$

$0 \rightarrow -3$  is a root

$x + 3$  is a factor

d) check  $\frac{1}{2}$

$$4\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right) - 3$$

$$1 + 2 - 3$$

$0 \leftarrow \frac{1}{2}$  is a root

$(2x - 1)$  is a factor

c)  $4x + 1 = 0$

$$x = -\frac{1}{4}$$

check  $\frac{6}{16} - \frac{11}{4} + 4$

not zero

∴  $4x + 1$  not a factor

19. Solve each equation.

a)  $x(2x - 3) - 2(3 + 2x) = -4(x + 1)$

b)  $3(x - 2)(x + 1) - 4 = 2(x - 1)^2$

$$a) 2x^2 - 3x - 6 - 4x = -4x - 4$$

$$2x^2 - 3x - 2 = 0$$

$$(2x + 1)(x - 2) = 0$$

$$x = -\frac{1}{2} \quad x = 2$$

$$b) 3(x^2 - x - 2) - 4 = 2(x^2 - 2x + 1)$$

$$3x^2 - 3x - 6 - 4 = 2x^2 - 4x + 2$$

$$x^2 + x - 12 = 0$$

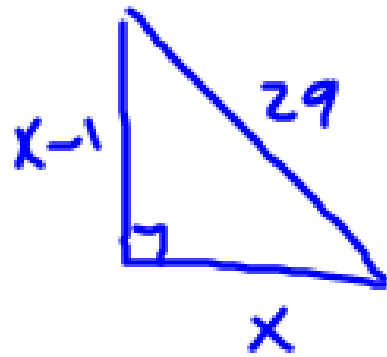
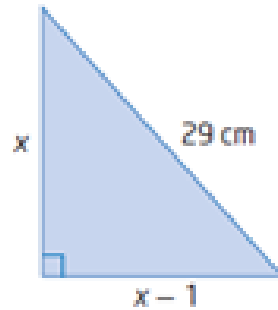
$$(x + 4)(x - 3) = 0$$

↓

↓

$$x = -4 \quad x = 3$$

20. The hypotenuse of a right triangle measures 29 cm. One leg is 1 cm shorter than the other. What are the lengths of the legs?



$$\begin{aligned} & \underline{(x-1)} \underline{(x-1)} \\ & x^2 - x - x + 1 \end{aligned}$$

$$x^2 + (x-1)^2 = 29^2$$

$$x^2 + \underline{x^2 - 2x + 1} = 29^2$$

$$2x^2 - 2x + 1 = 841$$

$$2x^2 - 2x - 840 = 0$$

$$x^2 - x - 420 = 0$$

$$(x-21)(x+20) = 0$$

↓

$$x = 21$$

↓

$$x = -20$$

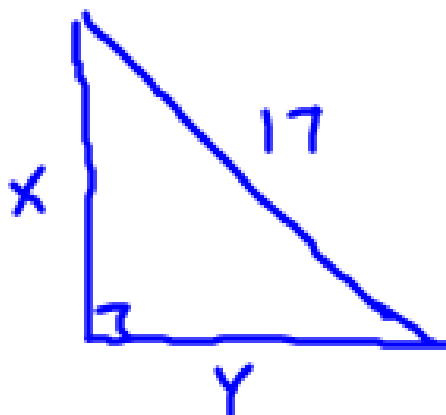
↓

$$x = 21$$

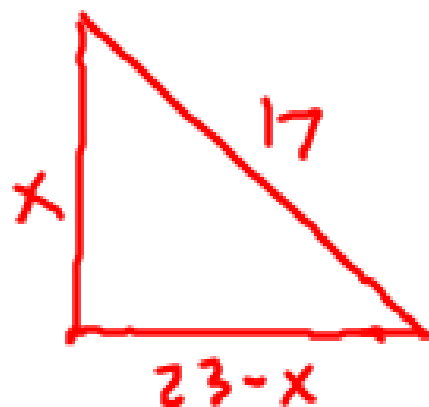
← negative lengths.

legs 20, 21 cm

21. A field is in the shape of a right triangle. The fence around the perimeter of the field measures 40 m. If the length of the hypotenuse is 17 m, find the length of the other two sides.



$$\begin{aligned} x + y + 17 &= 40 \\ x + y &= 23 \\ y &= 23 - x \end{aligned}$$



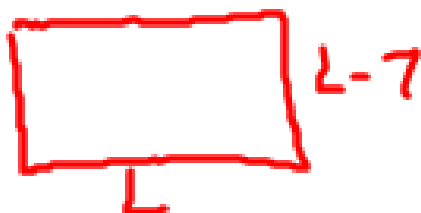
$$\begin{aligned} x^2 + (23 - x)^2 &= 17^2 \\ x^2 + x^2 - 46x + 529 &= 289 \\ 2x^2 - 46x + 240 &= 0 \\ x^2 - 23x + 120 &= 0 \\ (x - 8)(x - 15) &= 0 \\ \downarrow \quad \downarrow \\ x = 8 \quad x = 15 \end{aligned}$$

8 gives 8, 15  
15 gives 15, 8

lengths  
15 cm, 8 cm

22. The width of the top of a notebook computer is 7 cm less than the length. The surface area of the top of the notebook is  $690 \text{ cm}^2$ .

- Write an equation to represent the surface area of the top of the notebook computer.
- What are the dimensions of the top of the computer?



$$a) 690 = L(L-7)$$

$$b) 0 = L^2 - 7L - 690$$

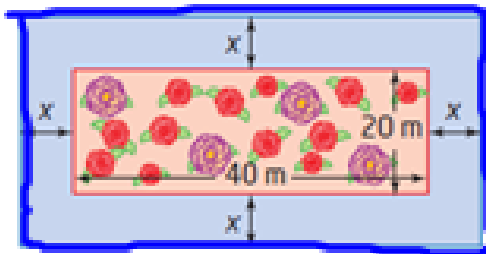
$$0 = (L-30)(L+23)$$

$$\begin{array}{cc} \downarrow & \downarrow \\ L=30 & L=-23 \text{ (discard)} \end{array}$$

$$30 \text{ cm} \times 23 \text{ cm}$$



23. Stephan plans to build a uniform walkway around a rectangular flower bed that is 20 m by 40 m. There is enough material to make a walkway that has a total area of 700 m<sup>2</sup>. What is the width of the walkway?



blue rectangle area  
 $(40+2x)(20+2x)$

blue walkway area

$$\underline{(40+2x)}(\underline{20+2x}) - (40)(20) = 700$$

$$4x^2 + 120x - 700 = 0$$

$$x^2 + 30x - 175 = 0$$

$$(x+35)(x-5) = 0$$

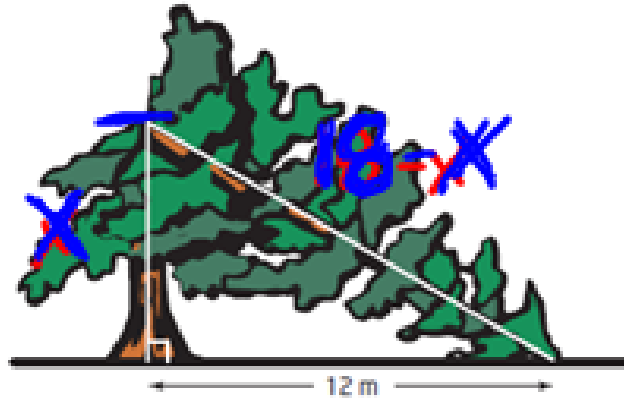
$$\downarrow$$
$$x = -35$$

discard

$$\downarrow$$
$$x = 5$$

$$\textcircled{5 \text{ m}}$$

24. An 18-m-tall tree is broken during a severe storm, as shown. The distance from the base of the trunk to the point where the tip touches the ground is 12 m. At what height did the tree break?



$$x^2 + 12^2 = (18 - x)^2$$

$$x^2 + 144 = x^2 - 36x + 324$$

$$36x = 180$$

$$x = 5$$

$$5\text{m}$$

25. The pressure difference,  $P$ , in newtons per square metre, above and below an airplane wing is described by the formula  $P = \left(\frac{1}{2}d\right)(v_1)^2 - \left(\frac{1}{2}d\right)(v_2)^2$ , where  $d$  is the density of the air, in kilograms per cubic metre;  $v_1$  is the velocity, in metres per second, of the air passing above; and  $v_2$  is the velocity, in metres per second, of the air passing below. Write this formula in factored form.

$$\textcircled{1} P = \frac{1}{2}d [v_1^2 - v_2^2] \quad \text{common factor}$$

$$\textcircled{1} P = \frac{1}{2}d (v_1 - v_2)(v_1 + v_2) \quad \text{difference of squares}$$

26. Carlos was asked to factor the trinomial  $6x^2 - 16x + 8$  completely. His work is shown below.

*Carlos's solution:*

$$\begin{aligned} & 6x^2 - 16x + 8 \\ &= 6x^2 - 12x - 4x + 8 \\ &= 6x(x - 2) - 4(x - 2) \\ &= (x - 2)(6x - 4) \end{aligned}$$

Is Carlos correct? Explain.

$(x-2)(6x-4)$  good so far!  
↑  
common factor

$$(x-2) \cdot 2(3x-2)$$

$$2(x-2)(3x-2)$$

27. Factor each expression.

a)  $3(2z + 3)^2 - 9(2z + 3) - 30$

b)  $16(m^2 - 4)^2 - 4(3n)^2$

c)  $\frac{1}{9}y^2 - \frac{1}{3}yx + \frac{1}{4}x^2$

d)  $-28\left(w + \frac{2}{3}\right)^2 + 7\left(3w - \frac{1}{3}\right)^2$

a)  $3a^2 - 9a - 30$

$3(a^2 - 3a - 10)$

$3(a - 5)(a + 2)$

$3(2z + 3 - 5)(2z + 3 + 2)$

$3(2z - 2)(2z + 5)$

$6(z - 1)(2z + 5)$

b) diff of squares

$$\left[4(m^2 - 4) - 2(3n)\right] \left[4(m^2 - 4) + 2(3n)\right]$$

c) perfect square

$$\left(\frac{1}{3}y - \frac{1}{2}x\right) \left(\frac{1}{3}y - \frac{1}{2}x\right)$$

d) common factor

$$-7 \left[4\left(w + \frac{2}{3}\right)^2 - \left(3w - \frac{1}{3}\right)^2\right]$$

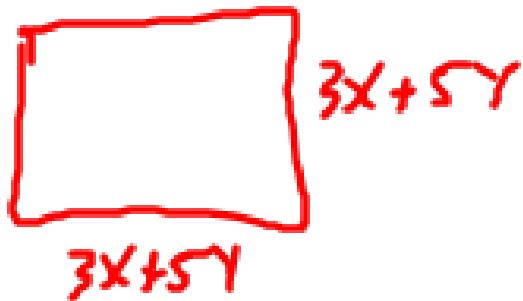
diff of squares

$$-7 \left[2\left(w + \frac{2}{3}\right) + \left(3w - \frac{1}{3}\right)\right] \left[2\left(w + \frac{2}{3}\right) - \left(3w - \frac{1}{3}\right)\right]$$

## Extend

28. A square has an area of  $(9x^2 + 30xy + 25y^2)$  square centimetres. What is an expression for the perimeter of the square?

$$(3x + 5y)(3x + 5y) \quad \text{perfect square.}$$



$$\begin{aligned} \text{perimeter} &= 4(3x + 5y) \\ &= 12x + 20y \end{aligned}$$

## Create Connections

30. Write a quadratic equation in standard form with the given root(s).

a)  $-3$  and  $3$

b)  $2$

c)  $\frac{2}{3}$  and  $4$

d)  $\frac{3}{5}$  and  $-\frac{1}{2}$

a)  $-3$  and  $3$  are roots  
 $(x+3)(x-3)$  are factors  
 $(x+3)(x-3) = 0$   
 $x^2 - 9 = 0$

b)  $2$  is a double/equal root  
 $(x-2)(x-2) = 0$   
 $x^2 - 4x + 4 = 0$

c)  $(x - \frac{2}{3})(x - 4) = 0$   
 $(3x - 2)(x - 4) = 0$   
 $3x^2 - 14x + 8 = 0$

d)  $x = \frac{3}{5}$   $x = -\frac{1}{2}$   
 $(5x - 3)(2x + 1) = 0$   
 $10x^2 - x - 3 = 0$

## 4.3 Solving quadratics by completing the square

ex1  $2x^2 + 3x - 7 = 0$

\* factoring: product  $-14$ , sum  $3$   
(not possible)

①  $2x^2 + 3x = 7$

move constant

②  $x^2 + \frac{3}{2}x = \frac{7}{2}$

divide by  $x^2$  coeff.

③  $x^2 + \frac{3}{2}x + \frac{9}{16} = \frac{7}{2} + \frac{9}{16}$

$\frac{3}{2} \rightarrow \frac{3}{4} \rightarrow \frac{9}{16}$

Complete the square

$\frac{1}{2}$  the  $x$  coeff. squared (add to both sides)

④  $(x + \frac{3}{4})^2 = \frac{65}{16}$

④ write as perfect square, simplify

⑤  $x + \frac{3}{4} = \pm \sqrt{\frac{65}{16}}$

⑤  $\sqrt{\quad}$

$x = -\frac{3}{4} \pm \frac{\sqrt{65}}{4}$

$x = \frac{-3 \pm \sqrt{65}}{4}$

$\frac{-3 + \sqrt{65}}{4}$  or  $\frac{-3 - \sqrt{65}}{4}$



## Practise

1. What value of  $c$  makes each expression a perfect square?

a)  $x^2 + x + c$

b)  $x^2 - 5x + c$

a)  $x^2 + x + \frac{1}{4}$

$(x + \frac{1}{2})^2$

$1 \rightarrow \frac{1}{2} \rightarrow \frac{1}{4}$

b)  $x^2 - 5x + \frac{25}{4}$

$(x - \frac{5}{2})^2$

$-5 \rightarrow -\frac{5}{2} \rightarrow \frac{25}{4}$

2. Complete the square to write each quadratic equation in the form  $(x + p)^2 = q$ .

a)  $2x^2 + 8x + 4 = 0$

b)  $-3x^2 - 12x + 5 = 0$

a)  $2x^2 + 8x = -4$

$x^2 + 4x = -2$

$x^2 + 4x + 4 = -2 + 4$

$(x+2)^2 = 2$

$x+2 = \pm\sqrt{2}$

$x = -2 \pm\sqrt{2}$

$4 \rightarrow 2 \rightarrow 4$

-2

$\frac{-6}{2} = -3$

b)  $-3x^2 - 12x = -5$   
 $x^2 + 4x = \frac{5}{3}$

$4 \rightarrow 2 \rightarrow 4$

$x^2 + 4x + 4 = \frac{5}{3} + 4$

$(x+2)^2 = \frac{17}{3}$

$x+2 = \pm\sqrt{\frac{17}{3}}$

$x = -2 \pm\sqrt{\frac{17}{3}}$

$x = -2 \pm \frac{\sqrt{17}}{\sqrt{3}}$

$x = -2 \pm \frac{\sqrt{51}}{3}$  ✓

$x = \frac{-6 \pm \sqrt{51}}{3}$  ✓

pg.240 #3

3. Write each equation in the form  $(x-p)^2 = q$

~~$(x-p)^2 = q$~~

a)  $x^2 - 12x + 9 = 0$

b)  $5x^2 - 20x - 1 = 0$

a)  $x^2 - 12x = -9$        $-12 \rightarrow -6 \rightarrow 36$

$$x^2 - 12x + 36 = -9 + 36$$

$$(x-6)^2 = 27$$

b)  $5x^2 - 20x = 1$

$$x^2 - 4x = \frac{1}{5}$$

$$x^2 - 4x + 4 = \frac{1}{5} + 4$$

$$(x-2)^2 = \frac{21}{5}$$

$$-4 \rightarrow \textcircled{-2} \rightarrow 4$$

pg.240 #4

4. Solve each quadratic equation.

Express your answers as exact roots.

a)  $x^2 = 64$

b)  $2s^2 - 8 = 0$

c)  $\frac{1}{3}t^2 - 1 = 11$

d)  $-y^2 + 5 = -6$

a)  $x^2 = 64$

$x = \pm 8$

b)  $2s^2 = 8$

$s^2 = 4$

$s = \pm 2$

c)  $\frac{1}{3}t^2 = 12$

$t^2 = 36$

$t = \pm 6$

d)  $-y^2 = -11$

$y^2 = 11$

$y = \pm \sqrt{11}$

pg.240 #5

5. Solve. Express your answers as exact roots.

a)  $(x - 3)^2 = 4$

b)  $(x + 2)^2 = 9$

a)  $(x - 3)^2 = 4$

$$x - 3 = \pm 2$$

$$x = 3 \pm 2$$

$$x = 5, 1$$

b)  $x + 2 = \pm 3$

$$x = -2 \pm 3$$

$$x = 1, -5$$

6. Solve each quadratic equation by completing the square. Express your answers as exact roots.

a)  $x^2 + 10x = -4$   
 $x^2 + 10x + 25 = -4 + 25$   $10 \rightarrow 5 \rightarrow 25$   
 $(x+5)^2 = 21$   
 $x+5 = \pm \sqrt{21}$   
 $x = -5 \pm \sqrt{21}$

b)  $x^2 - 8x = -13$   
 $x^2 - 8x + 16 = -13 + 16$   $-8 \rightarrow -4 \rightarrow 16$   
 $(x-4)^2 = 3$   
 $x-4 = \pm \sqrt{3}$   
 $x = 4 \pm \sqrt{3}$

a)  $x^2 + 10x + 4 = 0$

b)  $x^2 - 8x + 13 = 0$

c)  $3x^2 + 6x + 1 = 0$

$3x^2 + 6x = -1$   $2 \rightarrow 1 \rightarrow 1$   
 $x^2 + 2x = -\frac{1}{3}$   
 $x^2 + 2x + 1 = -\frac{1}{3} + 1$

$(x+1)^2 = \frac{2}{3}$

$x+1 = \pm \sqrt{\frac{2}{3}}$

$x = -1 \pm \frac{\sqrt{2}}{\sqrt{3}}$

$x = -1 \pm \frac{\sqrt{6}}{3}$

$x = \frac{-3 \pm \sqrt{6}}{3}$

$-1$   
 $-\frac{3}{3}$   
 $\frac{3}{3}$

7. Solve each quadratic equation by completing the square. Express your answers to the nearest tenth.

a)  $x^2 - 8x = 4$       $-8 \rightarrow \textcircled{-4} \rightarrow 16$

$$x^2 - 8x + 16 = 4 + 16$$

$$(x - 4)^2 = 20$$

$$x - 4 = \pm \sqrt{20}$$

$$x = 4 \pm \sqrt{20}$$

$\swarrow$       $\searrow$   
 $4 + \sqrt{20}$       $4 - \sqrt{20}$   
 $\ast x = 4 \pm 2\sqrt{5}$

$x = 8.5$       $x = -0.5$

$$x = \frac{2 + \sqrt{19}}{3}$$

$$x = \frac{2 - \sqrt{19}}{3}$$

$$x = \frac{2 \pm \sqrt{19}}{3}$$

a)  $x^2 - 8x - 4 = 0$

b)  $-3x^2 + 4x + 5 = 0$

$$-3x^2 + 4x = -5$$

$$x^2 - \frac{4}{3}x = \frac{5}{3}$$

$$x^2 - \frac{4}{3}x + \frac{4}{9} = \frac{5}{3} + \frac{4}{9}$$

$$\left(x - \frac{2}{3}\right)^2 = \frac{19}{9}$$

$$x - \frac{2}{3} = \pm \sqrt{\frac{19}{9}}$$

$$x = \frac{2}{3} \pm \frac{\sqrt{19}}{3}$$

$$x = \frac{2 \pm \sqrt{19}}{3}$$

Today: finish 1,2,3,5,6,7, and do 8,9,10 pgs 240-241

Tomorrow: do 11-17