Chapter 3 Review Page 198 Question 1

a) The graph of $f(x) = (x + 6)^2 - 14$ will have the same shape as the graph of $f(x) = x^2$, since $a = 1$. Since $p = -6$ and $q = -14$, this represents a horizontal translation of 6 units to the left and a vertical translation of 14 units down relative to the graph of $f(x) = x^2$.
The vertex is located at $(-6, -14)$.
The equation of the axis of symmetry is $x = -6$.
The parabola opens upward.
The minimum value is -14.
The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \geq -14, y \in R\}$.

b) The graph of $f(x) = -2x^2 + 19$ will have a shape that is narrower than the graph of $f(x) = x^2$ and be reflected in the x-axis, since $a < -1$. Since $p = 0$ and $q = 19$, this represents a vertical translation of 19 units up relative to the graph of $f(x) = x^2$.
The vertex is located at $(0, 19)$.
The equation of the axis of symmetry is $x = 0$.
The parabola opens downward.
The maximum value is 19.
The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \leq 19, y \in R\}$.

c) The graph of $f(x) = \frac{1}{5}(x - 10)^2 + 100$ will have a shape that is wider than the graph of $f(x) = x^2$, since $0 < a < 1$. Since $p = 10$ and $q = 100$, this represents a horizontal translation of 10 units to the right and a vertical translation of 100 units up relative to the graph of $f(x) = x^2$.
The vertex is located at $(10, 100)$.
The equation of the axis of symmetry is $x = 10$.
The parabola opens upward.
The minimum value is 100.
The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \geq 100, y \in R\}$.

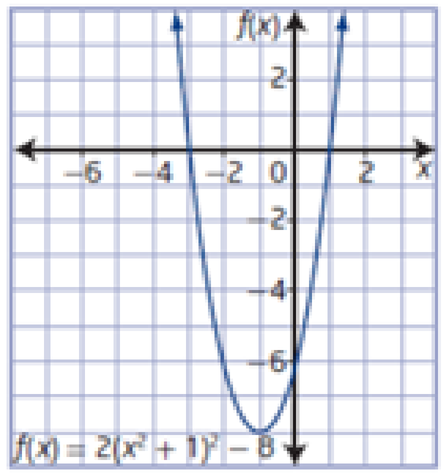
d) The graph of $f(x) = -6(x - 4)^2$ will have a shape that is narrower than the graph of $f(x) = x^2$ and be reflected in the x-axis, since $a < -1$. Since $p = 4$ and $q = 0$, this represents a horizontal translation of 4 units to the right relative to the graph of $f(x) = x^2$.
The vertex is located at (4, 0).
The equation of the axis of symmetry is $x = 4$.
The parabola opens downward.
The maximum value is 0.
The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \leq 0, y \in \mathrm{R}\}$.

Chapter 3 Review Page 198 Question 2

a) For $f(x) = 2(x + 1)^2 - 8$, $a = 2$, $p = -1$, and $q = -8$.
To sketch the graph of $f(x) = 2(x + 1)^2 - 8$, transform the graph of $f(x) = x^2$ by

- multiplying the y-values by a factor of 2
- translating 1 unit to the left and 8 units down

vertex: (–1, –8)
axis of symmetry: $x = -1$
domain: $\{x \mid x \in \mathrm{R}\}$
range: $\{y \mid y \geq -8, y \in \mathrm{R}\}$
x-intercepts: –3 and 1
y-intercept: –6

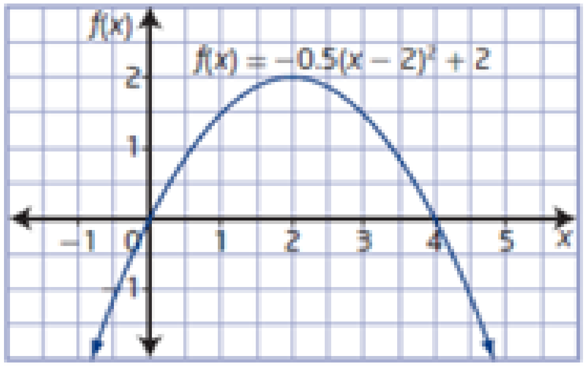


b) For $f(x) = -0.5(x - 2)^2 + 2$, $a = -0.5$, $p = 2$, and $q = 2$.

To sketch the graph of $f(x) = -0.5(x - 2)^2 + 2$, transform the graph of $f(x) = x^2$ by

- multiplying the y-values by a factor of 0.5
- reflecting in the x-axis
- translating 2 units to the right and 2 units up

vertex: (2, 2)
axis of symmetry: $x = 2$
domain: $\{x \mid x \in R\}$
range: $\{y \mid y \leq 2, y \in R\}$
x-intercepts: 0 and 4
y-intercept: 0



Chapter 3 Review Page 198 Question 3

a) For $y = -3(x - 5)^2 + 20$, $a = -3$, $p = 5$, and $q = 20$.
The vertex is located at (5, 20), which is above the x-axis. The graph opens downward, since $a < 0$. So, there are two x-intercepts.

b) Since the range is $\{y \mid y \geq 0, y \in R\}$, the vertex is located on the x-axis. So, there is one x-intercept.

c) For $y = 9 + 3x^2$, $a = 3$, $p = 0$, and $q = 9$.
The vertex is located at (0, 9), which is above the x-axis. The graph opens upward, since $a > 0$. So, there are no x-intercepts.

d) Given a vertex at (–4, –6), the parabola either has no x-intercepts if it opens downward or two x-intercepts if it opens upward.

Chapter 3 Review Page 198 Question 4

a) vertex at (0, 0), passing through the point (20, −150)
Since $p = 0$ and $q = 0$, the function is of the form $y = ax^2$.
Substitute the coordinates of the given point to find a.

$$-150 = a(20)^2$$
$$-150 = 400a$$
$$a = -\frac{3}{8}$$

The quadratic function in vertex form with the given characteristics is $y = -\frac{3}{8}x^2$.

b) vertex at (8, 0), passing through the point (2, 54)
Since $p = 8$ and $q = 0$, the function is of the form $y = a(x - 8)^2$.
Substitute the coordinates of the given point to find a.

$$54 = a(2 - 8)^2$$
$$54 = 36a$$
$$a = \frac{3}{2}$$

The quadratic function in vertex form with the given characteristics is $y = \frac{3}{2}(x - 8)^2$.

c) minimum value of 12 at $x = -4$ and y-intercept of 60
Since $p = -4$ and $q = 12$, the function is of the form $y = a(x + 4)^2 + 12$.
Substitute the coordinates of the y-intercept to find a.

$$60 = a(0 + 4)^2 + 12$$
$$48 = 16a$$
$$a = 3$$

The quadratic function in vertex form with the given characteristics is $y = 3(x + 4)^2 + 12$.

d) x-intercepts of 2 and 7 and maximum value of 25

The x-coordinate is halfway between the x-intercepts. So, $p = 4.5$.

Since $p = 4.5$ and $q = 25$, the function is of the form $y = a(x - 4.5)^2 + 25$.

Substitute the coordinates of one of the x-intercepts to find a.

$$0 = a(2 - 4.5)^2 + 25$$
$$-25 = 6.25a$$
$$a = -4$$

The quadratic function in vertex form with the given characteristics is $y = -4(x - 4.5)^2 + 25$.

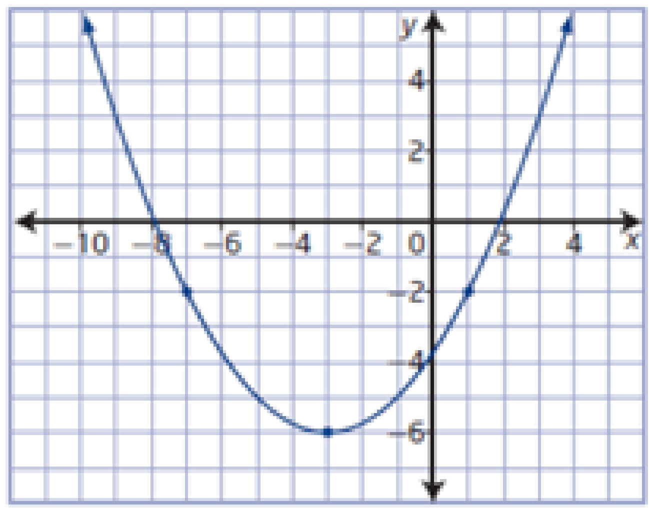
Chapter 3 Review Page 198 Question 5

a) Since the vertex is located at (–3, –4), $p = -3$ and $q = -6$.
So, the function is of the form $y = a(x + 3)^2 - 6$. Substitute (1, –2) and solve for a.

$$-2 = a(1 + 3)^2 - 6$$
$$4 = 16a$$
$$a = \frac{1}{4}$$

The quadratic function in vertex form is

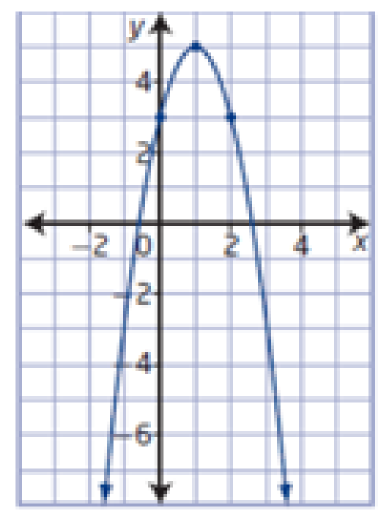
$$y = \frac{1}{4}(x + 3)^2 - 6.$$



b) Since the vertex is located at (1, 5), $p = 1$ and $q = 5$.
So, the function is of the form $y = a(x - 1)^2 + 5$. Substitute (0, 3) and solve for a.

$$3 = a(0 - 1)^2 + 5$$
$$3 = a + 5$$
$$a = -2$$

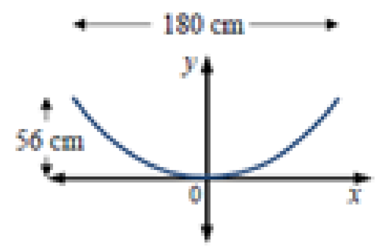
The quadratic function in vertex form is $y = -2(x - 1)^2 + 5$.



Chapter 3 Review Page 198 Question 6

Answers may vary. Example:

Choose the location of the origin to be the lowest point in the centre of the mirror. Let x and y represent the horizontal and vertical distances from the low point of the mirror, respectively.



Then, the vertex is at (0, 0) and the quadratic function is of the form $y = ax^2$. From the diagram, another point on the parabola is (90, 56).

Use the coordinates of this point to find a.

$56 = a(90)^2$

$56 = 8100a$

$a = \frac{14}{2025}$

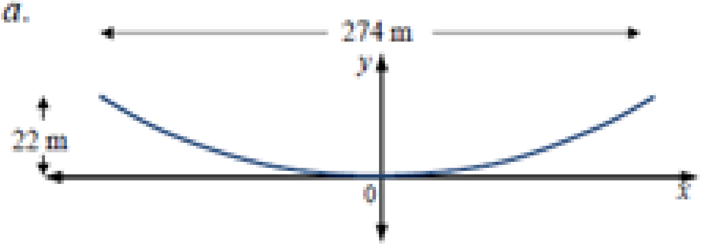
A quadratic function that represents the cross-sectional shape is $y = \frac{14}{2025}x^2$.

Chapter 3 Review Page 199 Question 7

a) i) The location of the origin is the lowest point in the centre of the cables.

Let x and y represent the horizontal and vertical distances from the low point of the cables, respectively. Then, the vertex is at (0, 0) and the quadratic function is of the form $y = ax^2$. From the diagram, another point on the parabola is (137, 22).

Use the coordinates of this point to find a.



$22 = a(137)^2$

$22 = 18\,769a$

$a = \frac{22}{18\,769}$

A quadratic function that represents the shape of the cables is $y = \frac{22}{18\,769}x^2$.

ii) The location of the origin is the point on the water's surface directly below the minimum point of the cables. Then the vertex is at (0, 30).
Let x and y represent the horizontal and vertical distances, respectively. The quadratic function is of the form $y = ax^2 + 30$. Another point on the parabola is (137, 52). Use the coordinates of this point to find a.

$52 = a(137)^2 + 30$

$22 = 18\,769a$

$$a = \frac{22}{18\,769}$$

A quadratic function that represents the shape of the cables is $y = \frac{22}{18\,769}x^2 + 30$.

iii) The location of the origin is the base of the tower on the left. Then the vertex is at (137, 30).
Let x and y represent the horizontal and vertical distances, respectively. The quadratic function is of the form $y = a(x - 137)^2 + 30$. Another point on the parabola is (0, 52). Use the coordinates of this point to find a.

$52 = a(0 - 137)^2 + 30$

$22 = 18\,769a$

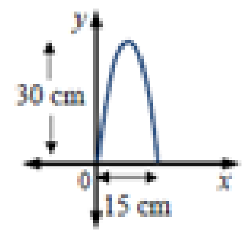
$$a = \frac{22}{18\,769}$$

A quadratic function that represents the shape of the cables is $y = \frac{22}{18\,769}(x - 137)^2 + 30$.

b) Answers may vary. Example: The function will change as the seasons change with the heat or cold changing the length of the cable.

Chapter 3 Review Page 199 Question 8

The location of the origin is at the point from which the flea jumped. Let x and y represent the horizontal and vertical distances, respectively. Then, the vertex is at (7.5, 30) and the quadratic function is of the form $y = a(x - 7.5)^2 + 30$. From the diagram, another point on the parabola is (0, 0).



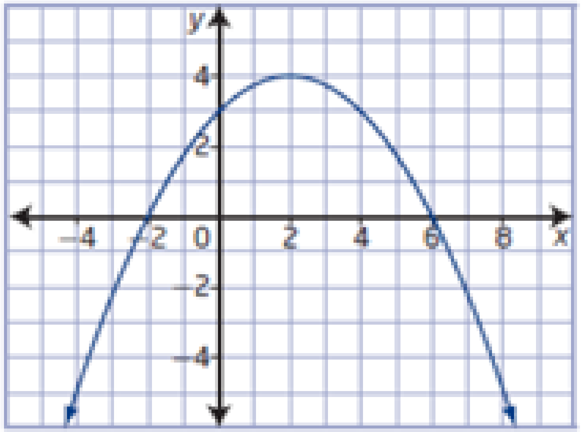
Use the coordinates of this point to find a.

$$0 = a(0 - 7.5)^2 + 30$$
$$-30 = 56.25a$$
$$a = -\frac{8}{15}$$

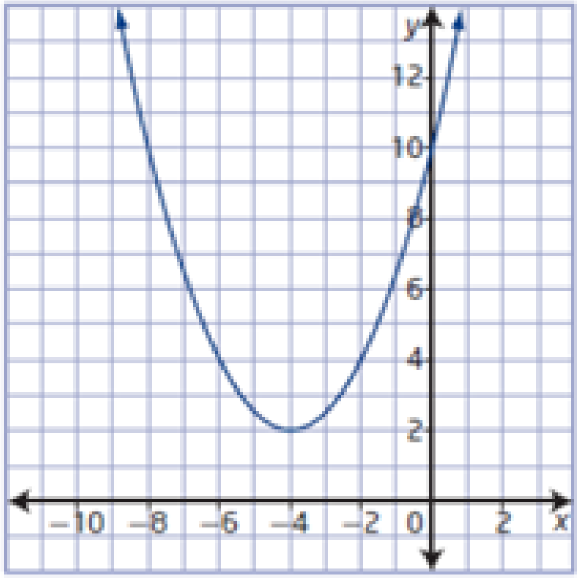
A quadratic function that represents the path of the flea is $y = -\frac{8}{15}(x - 7.5)^2 + 30$.

Chapter 3 Review   Page 199   Question 9

a) The coordinates of the vertex are (2, 4).
The equation of the axis of symmetry is $x = 2$.
The graph has a maximum value of 4, since the parabola opens downward.
The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \leq 4, y \in R\}$.
The x-intercepts are –2 and 6, and the y-intercept is 3.



b) The coordinates of the vertex are (–4, 2).
The equation of the axis of symmetry is $x = -4$.
The graph has a minimum value of 2, since the parabola opens upward.
The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \geq 2, y \in R\}$.
There are no x-intercepts, and the y-intercept is 10.



Chapter 3 Review Page 199 Question 10

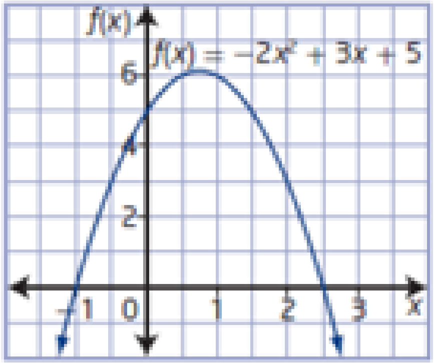
a) Expand $y = 7(x + 3)^2 - 41$.
$y = 7(x^2 + 6x + 9) - 41$
$y = 7x^2 + 42x + 63 - 41$
$y = 7x^2 + 42x + 22$
The function $y = 7(x + 3)^2 - 41$ is quadratic, since when expanded it is a polynomial of degree two.

b) Expand $y = (2x + 7)(10 - 3x)$.
$y = 20x - 6x^2 + 70 - 21x$
$y = -6x^2 - x + 70$
The function $y = (2x + 7)(10 - 3x)$ is quadratic, since when expanded it is a polynomial of degree two.

Chapter 3 Review Page 199 Question 11

a)

x	$f(x) = -2x^2 + 3x + 5$
−1	$f(-1) = -2(-1)^2 + 3(-1) + 5$ $= 0$
0	$f(0) = -2(0)^2 + 3(0) + 5$ $= 5$
1	$f(1) = -2(1)^2 + 3(1) + 5$ $= 6$
2	$f(2) = -2(2)^2 + 3(2) + 5$ $= 3$
3	$f(3) = -2(3)^2 + 3(3) + 5$ $= -4$

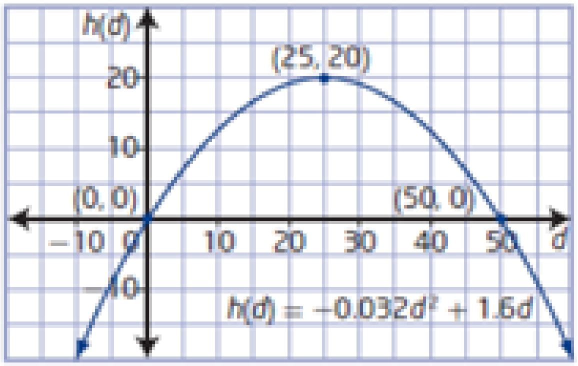


vertex: (0.75, 6.125)
axis of symmetry: $x = 0.75$
opens downward
maximum value: 6.125
domain: $\{x \mid x \in R\}$
range: $\{y \mid y \leq 6.125, y \in R\}$
x-intercepts: −1 and 3
y-intercept: 5

b) Answers may vary. Example: The vertex is the highest point on the parabola. The axis of symmetry is defined by the x-coordinate of the vertex. Since $a < 0$, the graph opens downward. The maximum value is the y-coordinate of the vertex. The domain is all real numbers. The range is less than or equal to the maximum value. The x-intercepts are where the graph crosses the x-axis, and the y-intercept is where the graph crosses the y-axis.

Chapter 3 Review Page 200 Question 12

a) Model the path of the soccer ball with a graph.



c) The ball hits the ground 50 m downfield.

d) Since both height and distance must be positive, the domain is $\{x \mid 0 \leq x \leq 50, x \in \mathbb{R}\}$ and the range is $\{y \mid 0 \leq y \leq 20, y \in \mathbb{R}\}$.

b) The maximum height of the ball is 20 m when the ball is 25 m downfield.

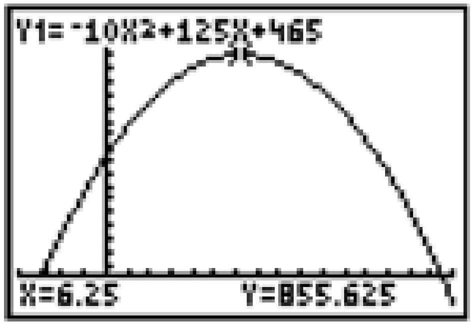
Chapter 3 Review Page 200 Question 13

a) Create a function model for the area, A.

$A = (31 - 2x)(5x + 15)$

$A = 155x + 465 - 10x^2 - 30x$

$A = -10x^2 + 125x + 465$



b) Use a graphing calculator to graph the function with window settings of x: [−4, 16, 1] and y: [−100, 1000, 50].

c) The portion of the graph above the x-axis represents the possible areas for the rectangle. So, the x-intercepts give the possible range of x-values that produce those areas.

d) The function has a maximum value and a minimum value (the minimum is 0 because of the context—it is an area).

e) The vertex gives the maximum area and the x-value for which it occurs.

f) The domain is $\{x \mid -3 \leq x \leq 15.5, x \in \mathbb{R}\}$ and the range is $\{A \mid 0 \leq A \leq 855.625, A \in \mathbb{R}\}$. The domain represents the values for x that will produce dimensions of a rectangle. The range represents the possible values of the area of the rectangle.

Chapter 3 Review Page 200 Question 14

a) Complete the square to write $y = x^2 - 24x + 10$ in vertex form.

$y = x^2 - 24x + 10$
$y = (x^2 - 24x) + 10$
$y = (x^2 - 24x + 144 - 144) + 10$
$y = (x^2 - 24x + 144) - 144 + 10$
$y = (x - 12)^2 - 134$

Expand $y = (x - 12)^2 - 134$ to verify the two forms are equivalent.

$y = (x - 12)^2 - 134$
$y = (x^2 - 24x + 144) - 134$
$y = x^2 - 24x + 10$

b) Complete the square to write $y = 5x^2 + 40x - 27$ in vertex form.

$y = 5x^2 + 40x - 27$
$y = 5(x^2 + 8x) - 27$
$y = 5(x^2 + 8x + 16 - 16) - 27$
$y = 5[(x^2 + 8x + 16) - 16] - 27$
$y = 5[(x + 4)^2 - 16] - 27$
$y = 5(x + 4)^2 - 80 - 27$
$y = 5(x + 4)^2 - 107$

Expand $y = 5(x + 4)^2 - 107$ to verify the two forms are equivalent.

$y = 5(x + 4)^2 - 107$
$y = 5(x^2 + 8x + 16) - 107$
$y = 5x^2 + 40x + 80 - 107$
$y = 5x^2 + 40x - 27$

c) Complete the square to write $y = -2x^2 + 8x$ in vertex form.

$y = -2x^2 + 8x$
$y = -2(x^2 - 4x)$
$y = -2(x^2 - 4x + 4 - 4)$
$y = -2[(x^2 - 4x + 4) - 4]$
$y = -2[(x - 2)^2 - 4]$
$y = -2(x - 2)^2 + 8$

Expand $y = -2(x - 2)^2 + 8$ to verify the two forms are equivalent.

$y = -2(x - 2)^2 + 8$
$y = -2(x^2 - 4x + 4) + 8$
$y = -2x^2 + 8x - 8 + 8$
$y = -2x^2 + 8x$

d) Complete the square to write $y = -30x^2 - 60x + 105$ in vertex form.

$y = -30x^2 - 60x + 105$
$y = -30(x^2 + 2x) + 105$
$y = -30(x^2 + 2x + 1 - 1) + 105$
$y = -30[(x^2 + 2x + 1) - 1] + 105$
$y = -30[(x + 1)^2 - 1] + 105$
$y = -30(x + 1)^2 + 30 + 105$
$y = -30(x + 1)^2 + 135$

Expand $y = -30(x + 1)^2 + 135$ to verify the two forms are equivalent.

$y = -30(x + 1)^2 + 135$
$y = -30(x^2 + 2x + 1) + 135$
$y = -30x^2 - 60x - 30 + 135$
$y = -30x^2 - 60x + 105$

Chapter 3 Review Page 200 Question 15

Write $f(x) = 4x^2 - 10x + 3$ in vertex form.

$f(x) = 4x^2 - 10x + 3$
$f(x) = 4(x^2 - 2.5x) + 3$
$f(x) = 4(x^2 - 2.5x + 1.5625 - 1.5625) + 3$
$f(x) = 4[(x^2 - 2.5x + 1.5625) - 1.5625] + 3$
$f(x) = 4[(x - 1.25)^2 - 1.5625] + 3$
$f(x) = 4(x - 1.25)^2 - 6.25 + 3$
$f(x) = 4(x - 1.25)^2 - 3.25$

For $f(x) = 4(x - 1.25)^2 - 3.25$, $a = 4$, $p = 1.25$, and $q = -3.25$.

vertex: $(1.25, -3.25)$
axis of symmetry: $x = 1.25$
minimum value: -3.25
domain: $\{x \mid x \in \mathrm{R}\}$
range: $\{y \mid y \geq -3.25, y \in \mathrm{R}\}$

Chapter 3 Review   Page 200   Question 16

a) *Amy's solution*:

$y = -22x^2 - 77x + 132$

$y = -22(x^2 - 3.5x) + 132$

$y = -22(x^2 - 3.5x - 12.25 + 12.25) + 132$

$y = -22(x^2 - 3.5x - 12.25) - 269.5 + 132$

$y = -22(x - 3.5)^2 - 137.5$

There is an error in line 2. Amy incorrectly factored –22 from 77*x*. The result should be + 3.5*x*. There is also an error in line 3. Amy should have added and subtracted the square of half the coefficient of the *x*-term, not subtracted and added the square of the coefficient of the *x*-term.

The corrected solution is shown.

$y = -22x^2 - 77x + 132$

$y = -22(x^2 + 3.5x) + 132$

$y = -22(x^2 + 3.5x + 3.0625 - 3.0625) + 132$

$y = -22(x^2 + 3.5x + 3.0625) + 67.375 + 132$

$y = -22(x + 1.75)^2 + 199.375$

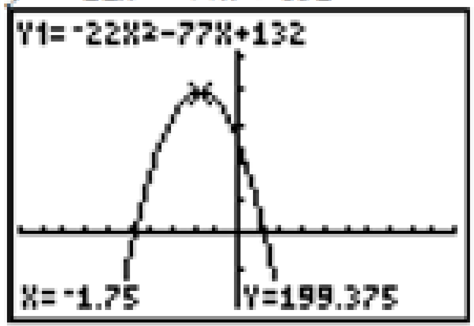
b) Answers may vary. Example: To verify an answer, either work backward to show the functions are equivalent or use technology to show the graphs of the functions are identical.

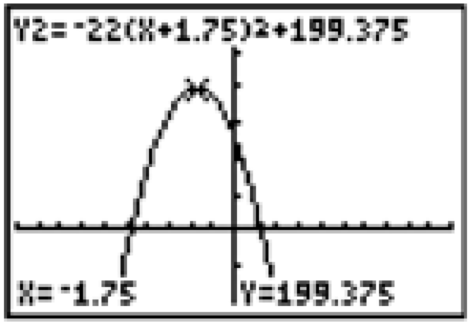
$y = -22(x + 1.75)^2 + 199.375$

$y = -22(x^2 + 3.5x + 3.0625) + 199.375$

$y = -22x^2 - 77x - 67.375 + 199.375$

$y = -22x^2 - 77x + 132$



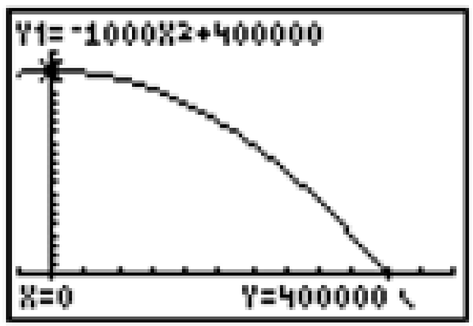


Chapter 3 Review Page 200 Question 17

a) Let n represent the number of price decreases. The new price is \$40 minus the number of price decreases times \$2, or $40 - 2n$.
The new number of coats sold is 10 000 plus the number of price decreases times 500, or $10\,000 + 500n$.
Let R represent the expected revenue, in dollars.
Revenue = (price)(number of sessions)
$R = (40 - 2n)(10\,000 + 500n)$
$R = 400\,000 - 1000n^2$
$R = -1000n^2 + 400\,000$

b) The function $R = -1000n^2 + 400\,000$ is in vertex form.
The maximum revenue the manager can expect is \$400 000 when a coat sells for $40 - 2(0)$, or \$40.

c) Use a graphing calculator to graph the function with window settings of
x: [–2, 24, 2] and
y: [–60 000, 500 000, 20 000].



d) The y-intercept represents the maximum revenue. The positive x-intercept indicates the number of price decreases that will produce revenue.

e) For the number of price decrease, the domain is $\{x \mid 0 \le x \le 20, x \in \mathrm{R}\}$ and the range is $\{y \mid 0 \le y \le 400\,000, y \in \mathrm{R}\}$.

f) Answers may vary. Example: Assume that the market research regarding price and number of coats sold holds true.