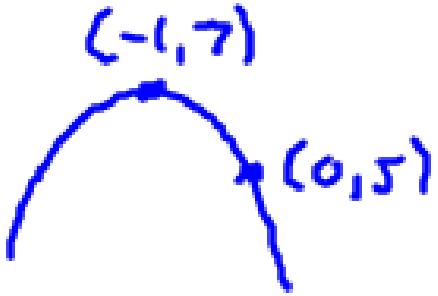
vertex form $y = a(x-p)^2 + q$

(3, 1)

$y = -2(x+1)^2 + 7$

concave down

(-1, 7)



Standard Form
(3.2)

$y = ax^2 + bx + c$

$y = 2x^2 - 4x + 7$

Concave up

y int (0, 7)

vertex? $x = \frac{-b}{2a} = \frac{4}{4} = 1$

$y = 2 - 4 + 7$

$y = 5$

(1, 5)

3.3 Completing the square

Completing the square is the process that leads to the shortcuts: vertex formula and quadratic formula

ex 1 $y = x^2 + 6x + 5$ (leading coeff. of 1)

$y - 5 = x^2 + 6x$ → move constant (if any)

$y - 5 + 9 = x^2 + 6x + 9$ → take $\frac{1}{2}$ the x coefficient, square it, add to both sides: $6 \to 3 \to 9$

$y + 4 = (x + 3)^2$ → write as perfect square

$y = (x + 3)^2 - 4$ vertex form

ex2 $y = 3x^2 - 12x + 9$ leading coeff $\neq 1$

$y - 9 = 3x^2 - 12x$ → move constant

$y - 9 = 3(x^2 - 4x)$ → factor out coeff. of x^2

$y - 9 + 12 = 3(x^2 - 4x + 4)$ → complete the square -2 4 * balance

$y + 3 = 3(x-2)^2$ → write as perfect square

$y = 3(x-2)^2 - 3$ → vertex form

ex 3 $f(x) = -5x^2 - 70x$

$f(x) = -5(x^2 + 14x)$ $\quad 7, 49$

$f(x) - 245 = -5(x^2 + 14x + 49)$

$f(x) - 245 = -5(x+7)^2$

$f(x) = -5(x+7)^2 + 245$

ex 4 $y = 4x^2 - 28x - 23$

$y + 23 = 4x^2 - 28x$

$y + 23 = 4(x^2 - 7x)$

$y + 23 + 49 = 4\left(x^2 - 7x + \frac{49}{4}\right)$

$y + 72 = 4\left(x - \frac{7}{2}\right)^2$

$y = 4\left(x - \frac{7}{2}\right)^2 - 72$

$-\frac{7}{2}$ $\quad \frac{49}{4}$

ex 5 $y = 5x^2 + 30x + 41$

$y - 41 = 5x^2 + 30x$

$y - 41 = 5(x^2 + 6x)$ 3 9

$y - 41 + 45 = 5(x^2 + 6x + 9)$

$y + 4 = 5(x+3)^2$

$y = 5(x+3)^2 - 4$

ex 6 redo ex 5 using vertex formula

$y = 5x^2 + 30x + 41$

$x = \frac{-b}{2a}$ $\frac{-30}{10} = -3$

$y = 5(-3)^2 + 30(-3) + 41$

$y = -4$

vertex $(-3, -4)$ $a = 5$

$y = 5(x+3)^2 - 4$

vertex formula (and quadratic formula) may be easier but PreCal students are responsible for the process of completing the square.

If a question does not specify completing the square you can use the shortcuts.

ex 7 (ex 4 pg 190)

"old" income = price × sessions

I = 10×400 ($4000)

for a $10 increase in price you lose 20 booked sessions

best price? max income?

let x = a $10 increase

"new" income

$I = (10+x)(400-20x)$

or $I = -20x^2 + 200x + 4000$

x-int: $10+x=0$, $x=-10$; $400-20x=0$, $20x=400$, $x=20$

$(-10,0)$ $(20,0)$

symmetry $\therefore$ vertex at $x=5$

$x=5$

-10 20

Income $\downarrow$ $(5, 4500)$

5 increases $\therefore$ $60

do 2, 3, 4, 6-12 pgs 192-194