1.5 Infinite Geometric Series

ex 1 Stand 20 m from a wall. Walk halfway towards wall. Repeat an infinite # of times.

$10 + 5 + 2.5 + 1.25 + \ldots \rightarrow 20$

$r = \frac{1}{2}$

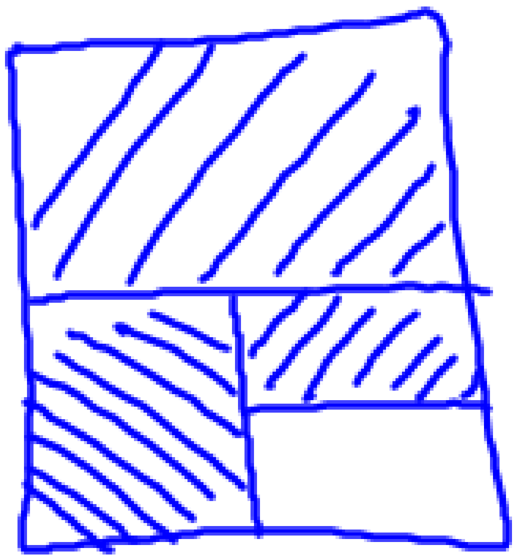
if $|r| < 1$ the sequence converges to 0

$r = -\frac{1}{2}$

ex2. Color/Shade half a piece of paper.

Shade $\frac{1}{2}$ of what's not shaded. Repeat....

Area = 1



shaded:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots = 1$$

$r = \frac{1}{2}$

$2+6+18+54+\ldots$ S_∞ ∞ $|r|>1$ <u>diverges</u>

$r=\underline{3}$

$9+3+1+\frac{1}{3}+\ldots$ $|r|<1$ Converges

$r=\frac{1}{3}$

$$S_\infty \quad \frac{9}{1-\frac{1}{3}}$$

$$\frac{9}{2/3}$$

$$\frac{27}{2}$$

$$\boxed{S_n = \frac{t_1(1-r^n)}{1-r}}$$

but if $|r| < 1$ and $n \to \infty$

$r^n \to$ $\left(\frac{1}{2}\right)^{1000} = 0?$

$\frac{1}{2}^{\infty} \to 0$

$$S_\infty = \frac{t_1}{1-r} \quad \text{for } |r| < 1$$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$

$$S_\infty = \frac{\frac{1}{2}}{1-\frac{1}{2}}$$

$= 1$

$10 + 5 + \ldots$

$$S_\infty = \frac{10}{1-\frac{1}{2}}$$

$= 20$

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a) $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} \ldots$

$r = -\frac{1}{3}$

$\left|-\frac{1}{3}\right| = \frac{1}{3}$

$S_\infty = \frac{1}{1 - -\frac{1}{3}}$

$\frac{1}{4/3}$ $= 3/4$

b) $2 - 4 + 8 - 16 \ldots$ $\infty, -\infty, \infty, -\infty$

$r = -2$

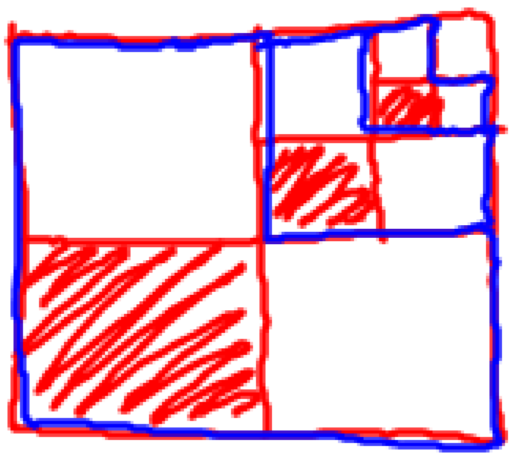
diverges

$2 + 4 + 8 + 16 \ldots$ ∞

$-2 - 4 - 8 - 16 \ldots$ $-\infty$

ex2 pg 62 $\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\ldots$

$r=\frac{1}{4}$



$$S_\infty=\frac{\frac{1}{4}}{1-\frac{1}{4}}$$

$$=\frac{\frac{1}{4}}{\frac{3}{4}}$$

$$=\frac{1}{3}$$

* use geometric series to prove that (a) $0.\overline{5} = \frac{5}{9}$ (b) $0.\overline{87} = \frac{87}{99}$

$= \frac{29}{33}$

$0.\overline{5} = 5/9$

a) 0.55555

$\frac{5}{10} + \frac{5}{100} + \frac{5}{1000} + \dots$

$r = \frac{1}{10}$

$S_\infty = \dfrac{\frac{5}{10}}{1 - \frac{1}{10}}$

$= \dfrac{5/10}{9/10} = 5/9$

$.\overline{123}$

$\frac{123}{999}$

3a from 1.5

b) 0.87878787

$\frac{87}{100} + \frac{87}{10000} + \frac{87}{1000000} + \dots$

$r = \frac{1}{100}$

$S_\infty = \dfrac{87/100}{1 - \frac{1}{100}}$

$= \dfrac{87/100}{99/100} = 87/99 \quad (29/33)$

$$t_n = t_1 + (n-1)d$$

$$S_n = \frac{n}{2}(t_1 + t_n)$$

$$= \frac{n}{2}(2t_1 + (n-1)d)$$

Arithmetic

$$t_n = t_1 r^{n-1}$$

$$S_n = \frac{t_1(r^n - 1)}{r-1} \quad \text{OR} \quad \frac{t_1(1-r^n)}{1-r}$$

$$S_n = \frac{rt_n - t_1}{r-1}$$

* if $|r| < 1$ for an infinite

$$S_\infty = \frac{t_1}{1-r}$$

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