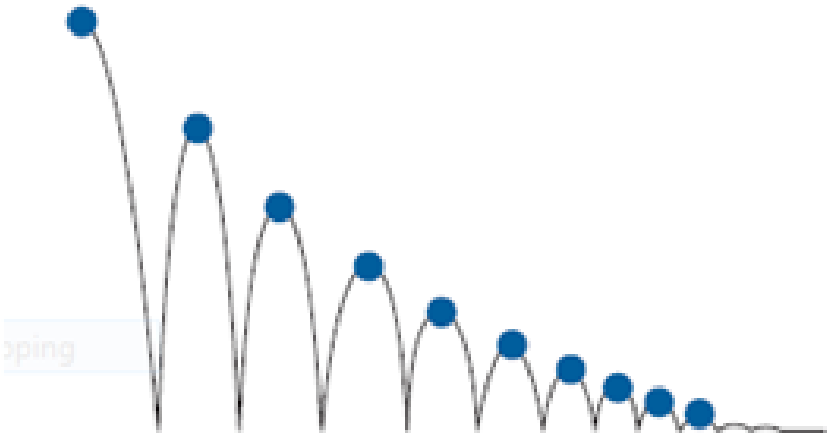
9. A ball is dropped from a height of 3.0 m. After each bounce it rises to 75% of its previous height.



a) Write the first term and the common ratio of the geometric sequence.

b) Write the general term for the sequence in part a).

c) What height does the ball reach after the 6th bounce?

d) After how many bounces will the ball reach a height of approximately 40 cm?

3, 2.25, ...

t_1 t_2

bounce 1

a) $t_1 = 3$

$r = 3/4$

b) $t_n = 3(3/4)^{n-1}$

c) 1 bounce = t_2

2 bounces = t_3

⋮

6 bounces: t_7

$t_7 = 3(3/4)^{7-1}$

$t_7 = 0.53$ m

53 cm

d) $0.4 = 3(3/4)^{n-1}$

$\frac{0.4}{3} = (\frac{3}{4})^{n-1}$

logs

↘ $Y_1 = 0.4/3$

$Y_2 = (3/4)^{n-1}$

calc intersect

$n-1 = \frac{\log(0.4/3)}{\log(.75)}$

$n-1 = 7$

$\therefore n = 8$ (7 bounces)

* Logarithms

for $a = b^{x}$

$$x = \frac{\log a}{\log b}$$

$2^x = 8$

$2^x = 2^3$

$\therefore x = 3$

$2^x = 16$

$2^x = 2^4$

$x = 4$

$2^x = 10$

$$x = \frac{\log 10}{\log 2}$$

$x = 3.32$

10. The colour of some clothing fades over time when washed. Suppose a pair of jeans fades by 5% with each washing.

a) What percent of the colour remains after one washing?

b) If $t_1 = 100$, what are the first four terms of the sequence?

c) What is the value of r for your geometric sequence?

d) What percent of the colour remains after 10 washings?

e) How many washings would it take so that only 25% of the original colour remains in the jeans? What assumptions did you make?

a) 95% remains.

b) 100, 95, 90.25, 85.7375 ...

c) .95

d) 1 wash t_2 2 wash t_3 10 wash t_{11}

$t_n = 100(.95)^{n-1}$

$t_{11} = 100(.95)^{11-1}$

$= 59.9\%$

e) $25 = 100(.95)^{n-1}$

$.25 = .95^{n-1}$

$\frac{\log .25}{\log .95} = n-1$

$n-1 = 27$

$n = 28$

t_{28} ∴ 27 washes.

19. The primary function for our kidneys is to filter our blood to remove any impurities. Doctors take this into account when prescribing the dosage and frequency of medicine. A person's kidneys filter out 18% of a particular medicine every two hours.

a) How much of the medicine remains after 12 h if the initial dosage was 250 mL? Express your answer to the nearest tenth of a millilitre.

removes 18%

∴ 82% remains.

$t_n = 250(.82)^n$

12 hrs ∴ $n = 6$

$t_n = 250(.82)^6 = 76 ml$

0 hr 2 hrs ... 12 hrs

250 250(.82)

t_1 t_2 t_7

b) When there is less than 20 mL left in the body, the medicine becomes ineffective and another dosage is needed. After how many hours would this happen?

$20 = 250(.82)^n$

$\frac{2}{25} = .82^n$

$n = \frac{\log(2/25)}{\log .82}$

$n = 12.7$ (2 hr)

25.5 hrs

* n: 2 hours.

1.4

Geometric Series

Developing a formula

$$S_n = t_1 + t_2 + t_3 + \cdots t_{n-1} + t_n$$

geometric

① $S_n = t_1 + t_1 r + t_1 r^2 + \cdots t_1 r^{n-1}$ → multiply by r

② $rS_n = \quad t_1 r + t_1 r^2 + \cdots \quad t_1 r^n$

subtract

$$S_n - rS_n = t_1 \qquad - t_1 r^n$$

$$S_n(1-r) = t_1(1-r^n)$$

$$\boxed{S_n = \frac{t_1(1-r^n)}{1-r}} \quad \text{OR} \quad \boxed{S_n = \frac{t_1(r^n-1)}{r-1}}$$

Example 1

Determine the Sum of a Geometric Series

Determine the sum of the first 10 terms of each geometric series.

a) $4 + 12 + 36 + \cdots$

$$S_{10} = \frac{4(3^{10}-1)}{3-1}$$

$$= 118096$$

example 2 Find S_8 for 243, 81, 27,...

$$S_8 = \frac{243\left(\left(\frac{1}{3}\right)^8 - 1\right)}{\left(\frac{1}{3} - 1\right)}$$

$$= \frac{3280}{9}$$

$$r = \frac{t_2}{t_1}$$

Example 2

Determine the Sum of a Geometric Series for an Unspecified Number of Terms

Determine the sum of each geometric series.

a) $\frac{1}{27} + \frac{1}{9} + \frac{1}{3} + \cdots + 729$

1 3 9 27 81 243

$r = 3$ $\therefore t_n = \frac{1}{27}(3)^{n-1}$

$729 = \frac{1}{27}(3)^{n-1}$

$19683 = 3^{n-1}$

$n - 1 = \frac{\log 19683}{\log 3}$

$n - 1 = 9$

$n = 10$

$S_{10} = \frac{\frac{1}{27}(3^{10} - 1)}{3 - 1}$

$= \frac{29524}{27}$

1093.48

$\frac{\frac{1}{9}}{\frac{1}{27}}$

$\frac{1}{9} \cdot \frac{27}{1}$

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