

1.3

Geometric Sequences

List the next 3 terms in the sequence 1, 2, 4, 8, 16, 32, 64, 128

geometric sequence: common ratio, r

$$t_1$$

$$t_2 = t_1 \cdot r$$

$$t_3 = t_1 \cdot r \cdot r$$

$$t_4 = t_1 \cdot r \cdot r \cdot r$$

$$t_n = t_1 (r)^{n-1}$$

$$r = \frac{t_4}{t_3} \quad \text{or} \quad \frac{t_2}{t_1}$$

$$\text{or} \quad \frac{t_n}{t_{n-1}} \quad \text{or} \quad \frac{t_{n+1}}{t_n}$$

Key Ideas

- A geometric sequence is a sequence in which each term, after the first term, is found by multiplying the previous term by a non-zero constant, r , called the common ratio.
- The common ratio of successive terms of a geometric sequence can be found by dividing any two consecutive terms, $r = \frac{t_n}{t_{n-1}}$.
- The general term of a geometric sequence is $t_n = t_1 r^{n-1}$
where t_1 is the first term
 n is the number of terms
 r is the common ratio
 t_n is the general term or n th term

Practise

1. Determine if the sequence is geometric.

If it is, state the common ratio and the general term in the form $t_n = t_1 r^{n-1}$.

a) 1, 2, 4, 8, ... **G**, $r=2$ $t_n = 2^{n-1}$

b) 2, 4, 6, 8, ... **not**

c) 3, -9, 27, -81, ... **G** $r=-3$ $t_n = 3(-3)^{n-1}$

d) 1, 1, 2, 4, 8, ... **not**

e) 10, 15, 22.5, 33.75, ... **G** $r=1.5$ $t_n = 10(1.5)^{n-1}$

f) -1, -5, -25, -125, ... **G** $r=5$ $t_n = -1(5)^{n-1}$

2. Copy and complete the following table for the given geometric sequences.

	Geometric Sequence	Common Ratio	6th Term	10th Term
a)	<u>6, 18, 54, ...</u>	3	$6(3)^5$ 1458	$6(3)^9$ 118098
b)	1.28, 0.64, 0.32, ...	$\frac{1}{2}$		$1.28(\frac{1}{2})^9$
c)	$\frac{1}{5}, \frac{3}{5}, \frac{9}{5}, \dots$	3		$\frac{1}{5}(3)^9$

$$t_n = 6(3)^{n-1}$$

$$6(3)^{29}$$

$$\frac{18}{6} = \frac{3}{1} \cdot \frac{3}{1} = 3$$

3. Determine the first four terms of each geometric sequence.

a) $t_1 = 2, r = 3$

b) $t_1 = -3, r = -4$

c) $t_1 = 4, r = -3$

d) $t_1 = 2, r = 0.5$

a) $2, 6, 18, 54 \dots$

b) $-3, 12, -48, 192 \dots$

c) $4, -12, 36, -108 \dots$

d) $2, 1, \frac{1}{2}, \frac{1}{4} \dots$

4. Determine the missing terms, t_2 , t_3 , and t_4 , in the geometric sequence in which $t_1 = 8.1$ and $t_5 = 240.1$.

$$t_1 = 8.1 \quad t_5 = 240.1$$

$$\begin{aligned} t_2 &= 8.1 \cdot r = 18.9 \\ t_3 &= 18.9 \cdot r = 44.1 \\ t_4 &= 44.1 \cdot r = 102.9 \end{aligned}$$

$$t_n = t_1(r)^{n-1}$$

$$240.1 = 8.1(r)^{5-1}$$

$$\frac{240.1}{8.1} = r^4$$

$$\left(\frac{240.1}{8.1}\right)^{\frac{1}{4}} = r$$

take 4th root

$$\frac{7}{3} = r$$

Arithmetic
 $t_2 = 7$
 $t_5 = 13$
 $d = 2$
 find t_{20}

5. Determine a formula for the n th term of each geometric sequence.

a) $r = 2, t_1 = 3$ $t_n = 3(2)^{n-1}$

b) $192, -48, 12, -3, \dots$ $t_n = 192(-0.25)^{n-1}$

c) $t_3 = 5, t_6 = 135$ $t_n = \frac{5}{9}(3)^{n-1}$

d) $t_1 = 4, t_{13} = 16\,384$

* $t_3 = 5$ $t_6 = 135$ $t_n = ?$

$$5 = t_1 \cdot r^2 \qquad \frac{135}{5} = \frac{t_1 \cdot r^5}{t_1 \cdot r^2}$$

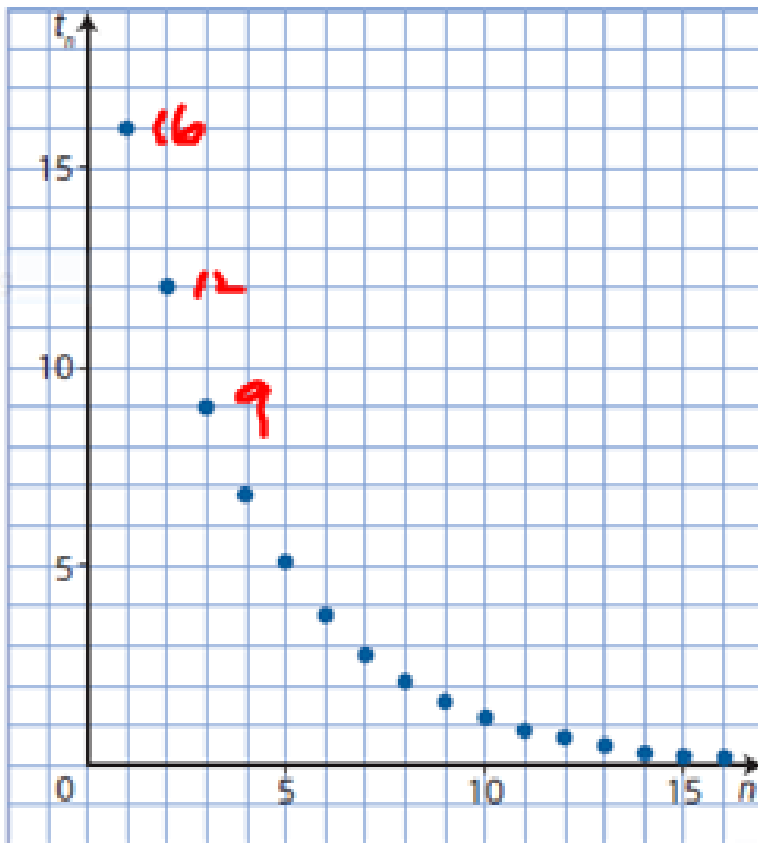
$$5 = t_1 \cdot 3^2$$

$$t_1 = \frac{5}{9}$$

$$27 = r^3$$

$$r = 3$$

8. The following graph illustrates a geometric sequence. List the first three terms for the sequence and state the general term that describes the sequence.



$$t_1, t_2, t_3$$
$$16, 12, 9$$

$$r = \frac{12}{16} = \frac{3}{4}$$

$$t_n = 16 \left(\frac{3}{4} \right)^{n-1}$$

23. If $x + 2$, $2x + 1$, and $4x - 3$ are three consecutive terms of a geometric sequence, determine the value of the common ratio and the three given terms.

$$A: t_2 - t_1 = t_3 - t_2$$

$$\frac{10}{2} = \frac{15}{3}$$

$$G: \frac{t_2}{t_1} = \frac{t_3}{t_2}$$

$$\frac{2x+1}{x+2} = \frac{4x-3}{2x+1}$$

$$4x^2 + 4x + 1 = 4x^2 + 5x - 6$$

$$7 = x$$

9, 15, 25

$$r = \frac{15}{9}, \frac{25}{15}, \text{ANS } \frac{5}{3}$$