

8.1 Solving Systems of equations.

Desmos.com demonstration.

ex 2 pg 428

$$4x - y + 3 = 0$$

$$2x^2 + 8x - y + 3 = 0$$

* "graphically"

$$4x + 3 = y$$

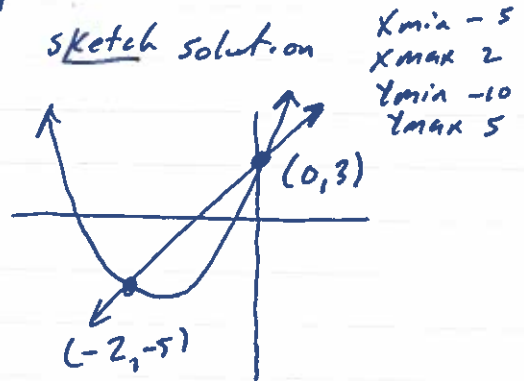
$$y_1 = 4x + 3$$

calc intersect

$$2x^2 + 8x + 3 = y$$

$$y_2 = 2x^2 + 8x + 3$$

sketch solution



b) verify (-2, -5)

$$4(-2) - (-5) + 3 = 0$$

$$-8 + 5 + 3 = 0$$

$$0 = 0 \checkmark$$

$$2(-2)^2 + 8(-2) - (-5) + 3 = 0$$

$$8 - 16 + 5 + 3 = 0$$

$$0 = 0 \checkmark$$

(0, 3)

$$4(0) - 3 + 3 = 0$$

$$0 - 3 + 3 = 0$$

$$0 = 0 \checkmark$$

$$2(0)^2 + 8(0) - 3 + 3 = 0$$

$$0 + 0 - 3 + 3 = 0$$

$$0 = 0 \checkmark$$

* algebraically

$$4x - y + 3 = 0 \quad 2x^2 + 8x - y + 3 = 0$$

$$\therefore 4x - y + 3 = 2x^2 + 8x - y + 3$$

$$0 = 2x^2 + 4x$$

$$0 = 2x(x + 2)$$

$$2x = 0 \quad x + 2 = 0$$

$$x = 0 \quad x = -2$$

$$(0, 3) \quad (-2, -5)$$

ex 3 pg 429

$$2x^2 - 16x - y = -35$$

$$2x^2 - 16x + 35 = y$$

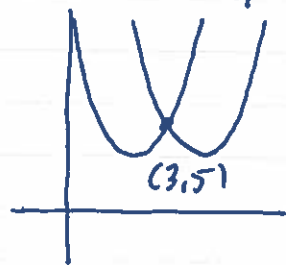
$$y_1 = 2x^2 - 16x + 35$$

$$2x^2 - 8x - y = -11$$

$$2x^2 - 8x + 11 = y$$

$$y_2 = 2x^2 - 8x + 11$$

Desmos.com



$x_{min} - 2$
 $x_{max} 10$
 $y_{min} - 2$
 $y_{max} 10$

calc intersect.

b) verify (3, 5)

$$2(3)^2 - 16(3) - 5 = -35$$

$$18 - 48 - 5 = -35$$

$$-35 = -35 \checkmark$$

$$2(3)^2 - 8(3) - 5 = -11$$

$$18 - 24 - 5 = -11$$

$$-11 = -11 \checkmark$$

* algebraically

$$y = 2x^2 - 16x + 35 \quad y = 2x^2 - 8x + 11$$

$$\therefore 2x^2 - 16x + 35 = 2x^2 - 8x + 11$$

$$-8x = -24$$

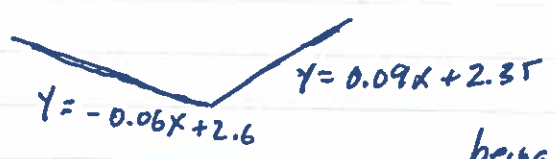
$$x = 3$$

$$y = 2(3)^2 - 8(3) + 11$$

$$y = 18 - 24 + 11$$

$$y = 5 \quad (3, 5)$$

ex 4 pg 430



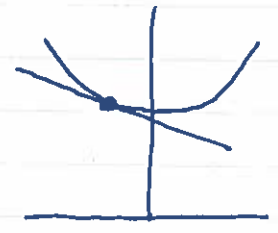
Desmos.com *all 3 at once

being replaced with $y = 0.0045x^2 + 2.8$

$$y_1 = -0.06x + 2.6$$

$$y_2 = 0.0045x^2 + 2.8$$

calc intersect
 $(-6.67, 3)$

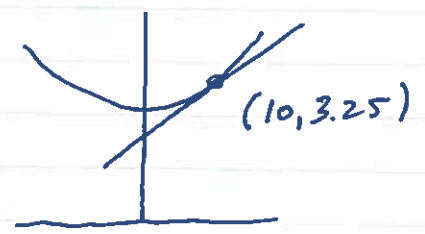


$x_{min} - 20$
 $x_{max} 20$
 $y_{min} 0$
 $y_{max} 5$

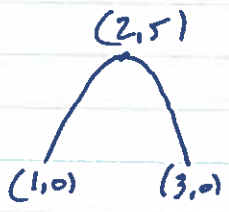
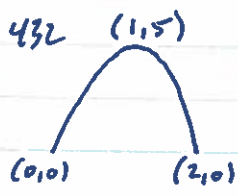
$$y_1 = 0.09x + 2.35$$

$$y_2 = 0.0045x^2 + 2.8$$

calc intersect



ex 5 pg 432



$$y = a(x-1)^2 + 5$$

$$(0,0) \quad 0 = a(1) + 5$$

$$-5 = a$$

a) $y_1 = -5(x-1)^2 + 5$

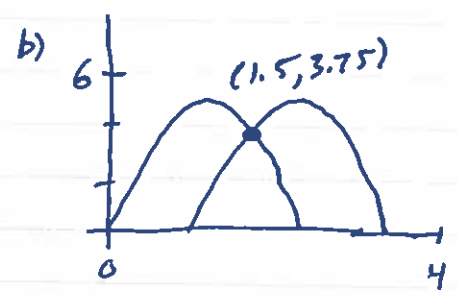
$$y = a(x-2)^2 + 5$$

$$(1,0) \quad 0 = a(1-2)^2 + 5$$

$$-5 = a$$

$$y_2 = -5(x-2)^2 + 5$$

Calc interest



(c) they high five at a height of 3.75 m, 1.5 seconds after the first person is launched.

- ① ¹⁻⁷ ~~pgs~~ pgs 435-436
- ② ~~pgs~~ pgs 436-439 8, 9, 10, 11, 13, 15, 17, 20

1 a) System A B and D launch from the same height. C launches from ground.
 b) the intersection is a point at which the distance away from the ramp, x , and the height, y , are equal.

$$2 \quad (0, -5) : \quad -5 = -(0)^2 + 4(0) - 5 \quad -5 = 0 - 5$$

$$\quad \quad \quad -5 = -5 \checkmark \quad \quad \quad -5 = -5 \checkmark$$

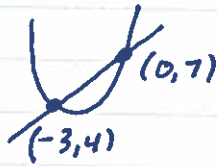
$$(3, -2) \quad -2 = -(3)^2 + 4(3) - 5 \quad -2 = 3 - 5$$

$$\quad \quad \quad -2 = -9 + 12 - 5 \quad -2 = -2 \checkmark$$

$$\quad \quad \quad -2 = -2 \quad \checkmark$$

- 3 a) linear-quadratic $(-4, 1)$ and $(-1, -2)$
- b) quadratic-quadratic no solution
- c) linear-quadratic $(1, -4)$

4a) $Y_1 = x + 7$
 $Y_2 = (x+2)^2 + 3$

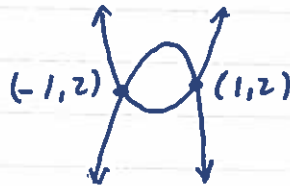


verify (0,7) (-3,4)
 $0+7=7 \checkmark$ $-3+7=4 \checkmark$
 $2^2+3=7 \checkmark$ $(-1)^2+3=4 \checkmark$

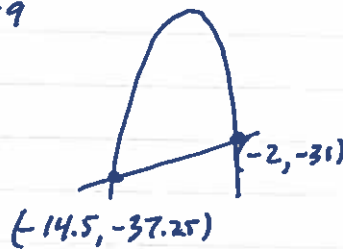
b) $Y_1 = -x + 5$
 $Y_2 = \frac{1}{2}(x-4)^2 + 1$



d) $Y_1 = 3 - x^2$
 $Y_2 = x^2 + 1$

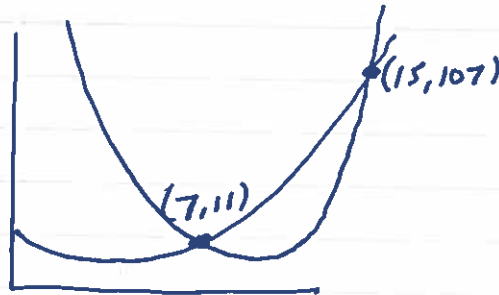


c) $Y_1 = -x^2 - 16x - 59$
 $-2Y_2 = 60 - x$
 $Y_2 = -30 + \frac{1}{2}x$



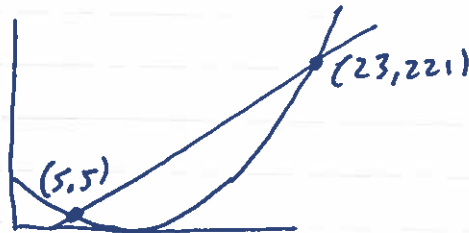
$X_{min} -40$
 $X_{max} 10$
 $Y_{min} -40$
 $Y_{max} 10$

e) $Y_1 = x^2 - 10x + 32$
 $Y_2 = 2x^2 - 32x + 137$



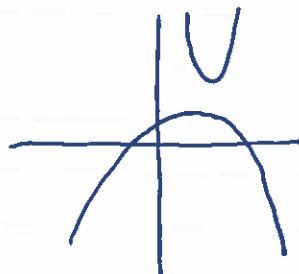
$X_{min} 0$
 $X_{max} 20$
 $Y_{min} 0$
 $Y_{max} 120$

5a) $Y_1 = x^2 - 16x + 60$
 $Y_2 = 12x - 55$



$X_{min} 0$
 $X_{max} 30$
 $Y_{min} 0$
 $Y_{max} 300$

b) $Y_1 = 3x^2 - 12x + 17$
 $Y_2 = -0.25x^2 + 0.5x + 1.75$

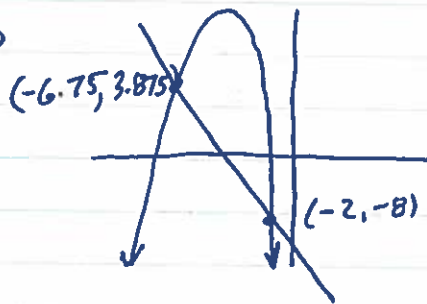


$-10 \leq x \leq 10$
 $-10 \leq y \leq 10$

No Solution

pg 436

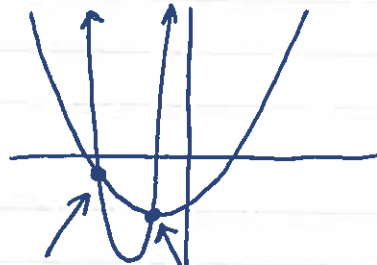
5c) $Y_1 = -2x^2 - 20x - 40$
 $Y_2 = -\frac{5}{2}x^2 - 13$



$-10 \leq x \leq 10$
 $-10 \leq y \leq 10$

5d) $n^2 + 2n - 7 = 2m$

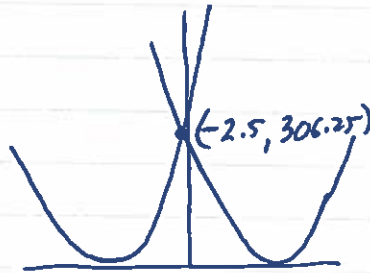
$Y_1 = \frac{1}{2}x^2 + x - \frac{7}{2}$
 $3n^2 + 12n + 6 = m$
 $Y_2 = 3x^2 + 12x + 6$



$-10 \leq x \leq 10$
 $-10 \leq y \leq 10$

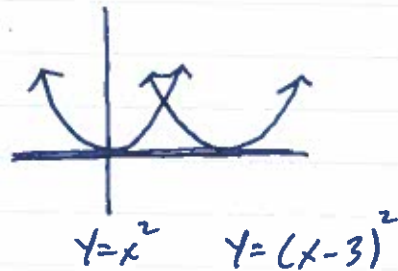
$(-3.21, -1.54)$ $(-1.18, -3.98)$

5e) $Y_1 = x^2 + 40x + 400$
 $Y_2 = x^2 - 30x + 225$

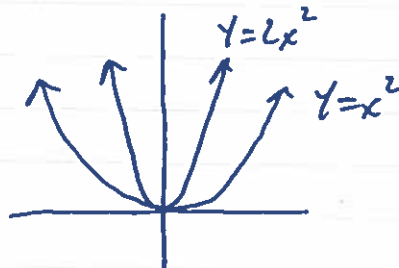


$-30 \leq x \leq 30$
 $0 \leq y \leq 400$

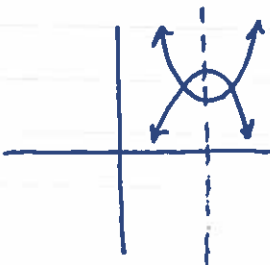
6. same "slope", different vertex



same vertex, different "slope"

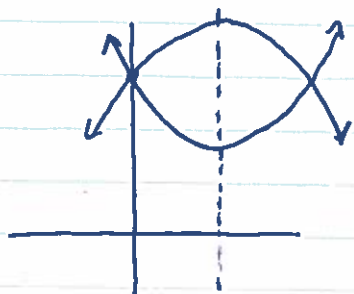


7a)



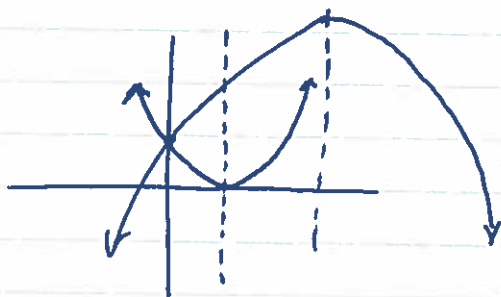
2 solutions
 same axis of symmetry.

7b)



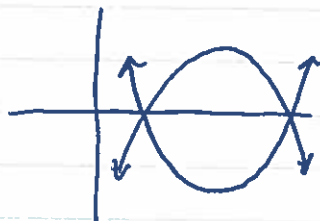
same axis of symmetry
same y intercept
2 solutions

7c)



2 solutions
same y intercept
different axis of symmetry

7d)



2 solutions
same x intercepts.

8a) $y = -3$ (no solutions)

b) $y = -2$ (1 solution)

c) $y = 1$ (2 solutions)

9, 10, 11, 13, 15, 17, 20

pg 436

#9 a) (100, 3300) (1000, 8000)

b) if he sells 100 - 1000 shirts he will not lose money

c) 500 shirts Revenue = 15000 Cost = 5500 Profit 9500
 ↑
 approximately

#10 $Y_1 = -0.04x + 3.9$

$Y_2 = 0.001x^2 - 0.04x + 3.9$

(0, 3.9)

$Y_1 = 0.03x + 2.675$

$Y_2 = 0.001x^2 - 0.04x + 3.9$

(35, 3.725)

* zoom fit

* Calc intersect

#11 a) $Y_1 = 1.16x^2$
 $Y_2 = 1.74(x-3)^2$

b) $0 \leq x \leq 20$ (accelerates for 20 seconds)

c) Calc intersect

(16.3, 310.0)

(1.65, 3.16)

↑
 discard. (after 1.65 seconds the 2nd car hasn't started)

d) after 16.3 seconds both cars are at the 310m mark (16.3 after car 1 starts)

#13

~~AND~~
~~OR~~

$x + y = 21$

$2x^2 - 15 = y$

(y is larger)

$Y_1 = 21 - x$

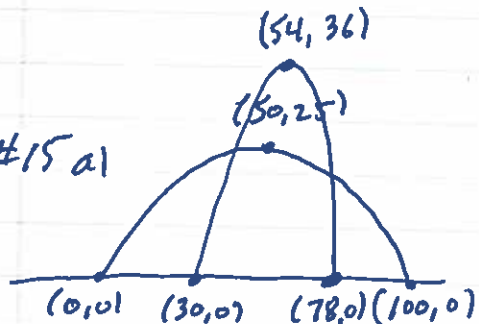
$Y_2 = 2x^2 - 15$

calc intersect (4, 17)

(-4.5, 25.5) discard, not integers

4 and 17

#15 a)



b) frog $y - 25 = a(x - 50)^2$

$y - 25 = -0.01(x - 50)^2$

grasshopper $y - 36 = a(x - 54)^2$

$y - 36 = -0.0625(x - 54)^2$

↑
 found a using quadreg

#17 a) linear (-1, 2)
 (2, 5)

slope $\frac{3}{3} = 1$

$y = 1x + b$ (2, 5)

$5 = 2 + b$

$3 = b$

$y = x + 3$

quadratic: let $(-1, 2)$ be vertex $y - 2 = a(x + 1)^2$ $(2, 5)$

$$5 - 2 = a(2 + 1)^2$$

$$3 = 9a$$

$$\frac{1}{3} = a$$

$$y = \frac{1}{3}(x + 1)^2 + 2$$

b) $y = \frac{1}{3}(x + 1)^2 + 2$
let $(2, 5)$ be vertex

$$y - 5 = a(x - 2)^2 \quad (-1, 2)$$

$$2 - 5 = a(-1 - 2)^2$$

$$-3 = 9a$$

$$-\frac{1}{3} = a$$

$$y = -\frac{1}{3}(x - 2)^2 + 5$$

c) get 1 equation for parabola, like $y = x^2 + 1$
and perform same operation on both sides

i.e. $y + 2x = x^2 + 1 + 2x$
 $y + 2x = (x^2 + 2x + 1)$

$$y = (x + 1)^2 - 2x$$

20 a) $y = x^2, y = x + 1 \quad \therefore x^2 = x + 1$
 $x^2 - x - 1 = 0 \quad b^2 - 4ac = 5 \quad \therefore 2 \text{ roots}$

b) $2x^2 + 3 = -2x - 5$
 $2x^2 + 2x + 8 = 0 \quad b^2 - 4ac = -60 \quad \therefore \text{no roots}$

c) $(x - 4)^2 + 1 = \frac{1}{3}(x - 4)^2 + 2$
 $\frac{2}{3}(x - 4)^2 = 1$
 $(x - 4)^2 = \frac{3}{2} \quad \therefore 2 \text{ roots}$

f) $(x + 5)^2 - 1 = x^2 + 10x + 24$
 $x^2 + 10x + 24 = x^2 + 10x + 24$
 \therefore infinite number of solutions

d) $2(x + 8)^2 - 9 = -2(x + 8)^2 - 9$
 $4(x + 8)^2 = 0$
 $(x + 8)^2 = 0 \quad \therefore 1 \text{ root}$

e) $2(x - 3)^2 + 1 = -2(x - 3)^2 - 1$
 $4(x - 3)^2 = -2 \quad (x - 3)^2 = -\frac{1}{2} \text{ no solution}$